

Using Java Geometry Expert as guide in the preparations for math contests

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We give an insight into Java Geometry Expert (JGEX) in use in a school context, focusing on the Austrian school system.

JGEX can offer great support in some classroom situations, especially for solving mathematical competition tasks.

Also, we discuss some limitations of the program.

The use of technical media in Austrian mathematics lessons is largely limited to GeoGebra.

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→ analytically consistent thinking, acquisition of maths skills, hypothetical-deductive thinking, inductive reasoning.

Olympic training courses in Austrian regions

Year 2022/23 (see <https://oemo.at/0eM0/Kurs/2023>)

<i>Region</i>	<i>Population (Jan. 2022)</i>	<i>Number of courses</i>
Burgenland	297583	0
Carinthia	564513	5
Lower Austria	1698796	7
Upper Austria	1505140	18
Salzburg	560710	2
Styria	1252922	33
Tyrol	764102	6
Vienna	1931593	14
Vorarlberg	401647	3
<i>Overall</i>	8977006	88

Preparation day for young learners

2 February 2023, Upper Austria, Linz, Johannes Kepler University



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Geometry problems to apply the inscribed angle theorem

A collection by Ralf Roupec, olympic trainer, region Freistadt, 2018

Aufgaben zum Randwinkelsatz

Ein nützlicher Satz zu Beginn

1) Folgende Aussagen sind äquivalent:

- $ABCD$ ist ein Sehnenviereck.
- Die Punkte A, B, C und D liegen auf einem Kreis.
- $\sphericalangle A + \sphericalangle C = 180^\circ$
- $\sphericalangle A + \sphericalangle D = 180^\circ$

Aufgaben und Fragestellungen zum Schärfer des geometrischen Blicks

- 2) Gegeben sei ein spitzwinkliges Dreieck ABC . Weiters seien D, E und F die Fußpunkte der Höhen h_a, h_b und h_c .



Fragestellungen zu dieser Figur

- In der oben dargestellten Figur gibt es sechs Sehnenvierecke. Gib sie an!
 - Zeige, dass H der Inkreismitelpunkt des Dreiecks DEF ist.
- 3) Die Diagonalen des Vierecks $WXYZ$ stehen zueinander normal. Weiters sei $\sphericalangle WZX = 30^\circ$, $\sphericalangle XWY = 40^\circ$ und $\sphericalangle WYZ = 50^\circ$.

Berechne die Winkel $\sphericalangle WZY$ und $\sphericalangle WXY$!



- 4) $ABCDE$ sei ein konvexes Fünfeck, mit der Eigenschaft, dass $BCDE$ ein Quadrat ist und dass $\sphericalangle BAE = 90^\circ$. Der Diagonalschnittpunkt des Quadrats werde mit O bezeichnet. Zeige, dass AO den Winkel $\sphericalangle BAE$ halbiert!

- 5) Es sei ABC ein spitzwinkliges Dreieck und E und F die Fußpunkte der Höhen h_b und h_c . M sei der Mittelpunkt der Seite BC .

- Zeige, dass ME und MF Tangenten an den Umkreis von AFE sind.
- Zeige, dass die zu BC parallele Gerade durch den Punkt A ebenfalls eine Tangente des Umkreises von AFE ist.

- 6) Es sei ABC ein spitzwinkliges Dreieck mit dem Umkreis k . Es sei X der Mittelpunkt des Bogens BC , der A nicht enthält. Die Punkte Y und Z sind analog definiert (vgl. Skizze)



Zeige, dass der Höhenschnittpunkt von XYZ der Inkreismitelpunkt von ABC ist.

- 7) (**BAMO 1999/2**) Es sei $O = (0|0)$, $A = (0|a)$ und $B = (b|0)$, wobei $0 < a < b$. k sei der Kreis mit dem Durchmesser AB und P ein beliebiger Punkt von k . Die Gerade PA schneide die x -Achse im Punkt Q .

Zeige, dass $\sphericalangle BQP = \sphericalangle BOP$.

Schwierigere Aufgaben

- 8) Gegeben sei ein spitzwinkliges Dreieck ABC mit $BC > CA$. Die Streckensymmetrale der Strecke AB schneide die Gerade BC in P und die Gerade CA in Q . Der Fußpunkt des von P auf die Gerade CA gefällten Lotes wird mit R , der Fußpunkt des von Q auf die Gerade BC gefällten Lotes wird mit S bezeichnet.

Zeige, dass die Punkte R, S und der Mittelpunkt M der Strecke AB auf einer Geraden liegen.

- 9) Umkreis von AFE ist. Sei ABC ein spitzwinkliges Dreieck mit dem Umkreis U und sei der Punkt T so gewählt, dass TA eine Tangente an den Umkreis ist und $\sphericalangle TCB = 90^\circ$. Sei weiters D ein Punkt auf der Seite BC mit der Eigenschaft, dass $TD \parallel AB$.

Zeige, dass die Gerade DU durch den Punkt A verläuft.

- 10) (**IMO Shortlist 2010**) Sei ABC ein spitzwinkliges Dreieck mit den Höhenfußpunkten D, E, F auf den Seiten BC, CA und AB . Einer der Schnittpunkte des Umkreises von ABC mit der Gerade EF sei P . Die Geraden BP und DF schneiden sich im Punkt Q .

Zeige: $AP = AQ$

- 11) (**Russland 1996**) $ABCD$ ist ein Sehnenviereck. Die Punkte E und F liegen auf der Seite BC , wobei E näher bei B liegt als F . Weiters ist bekannt, dass $\sphericalangle BAE = \sphericalangle CDF$ und $\sphericalangle EAF = \sphericalangle FDE$.

Zeige, dass $\sphericalangle FAC = \sphericalangle EDB$.

Geometry problems to apply the inscribed angle theorem

A collection by Ralf Roupec, olympic trainer, region Freistadt, 2018

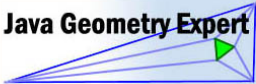
7) BAMO (Bay Area Mathematical Olympiad) 1999/2:

Set $O = (0, 0)$, $A = (0, a)$, $B = (0, b)$, where $0 < a < b$.
Let k be the circle with diameter AB , and let P be an arbitrary point on k . The line PA intersects the x -axis at point Q .

Show that $\angle BQP = \angle BOP$.

Java Geometry Expert (JGEX)

A free, open sourced framework to prove and discover planar geometry theorems



Java Geometry Expert

Java Geometry Expert 0.80
Last modified on 2023-04-29
Java version 11.0.20.1

Java Geometry Expert is free under GNU General Public License (GPL).
The user may download and distribute it freely.

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for more information, please visit: <https://github.com/kovzol/Java-Geometry-Expert>

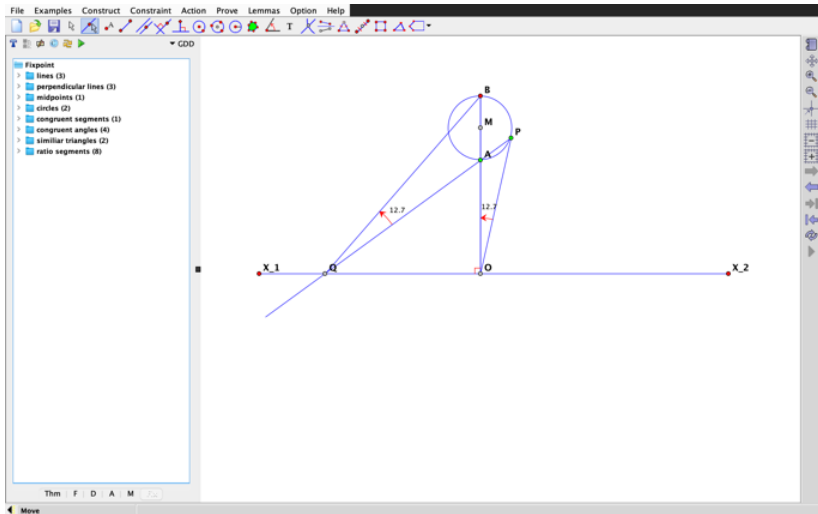
A solution of BAMO 1999/2 with JGEX

The screenshot displays the JGEX geometry software interface. The main workspace shows a geometric construction with points X_1 , X_2 , O , A , M , P , Q , and B . A horizontal line contains points X_1 , Q , O , and X_2 in order. A vertical line segment OB is drawn, with M as its midpoint. A circle is centered at M and passes through A and P . Lines connect Q to A and Q to B . Two angles are marked with red arcs and labeled 12.7 : one at vertex Q between lines QA and QB , and another at vertex O between lines OQ and OB . The left sidebar contains the following command log:

```
POINT X_1 X_2 B O A M P Q
ON_TLINE O B X_2 X_1
ON_LINE O X_1 X_2
ON_LINE A B O
MIDPOINT M A B
CIRCLE M A
ON_CIRCLE P M A
INTERSECTION_LL H X_1 X_2 A P
-----
Point: X_1, X_2, B, O, A, M, P, ...
OB : X_2X_1
O : on line X_1X_2
A : on line BO
M : midpoint(A B)
circle(M A)
P : on circle(M A)
Q = X_1X_2 = AP
```

At the bottom of the window, a status bar indicates: "Angle Select two lines to define their full-angle with a label".

A solution of BAMO 1999/2 with JGEX



A solution of BAMO 1999/2 with JGEX

The screenshot displays the JGEX software interface. The main workspace shows a geometric construction with the following elements:

- Horizontal line X_1X_2 with points Q and O on it.
- Vertical line segment BO perpendicular to X_1X_2 at O .
- A circle with center M and radius MB , tangent to BO at B .
- A line passing through Q and M , intersecting the circle at points A and P .
- Angles $\angle BQP$ and $\angle BOP$ are both marked as 12.7° .

The left-hand panel shows the following object list:

- Fixpoint
- lines (3)
- perpendicular lines (3)
- midpoints (1)
- circles (2)
- congruent segments (1)
- congruent angles (4)
 - 3. $\angle BAP = \angle APM = \angle X_1X_2BP$
 - 3. $\angle OBP = \angle BPM = \angle X_1QA$
 - 2. $\angle X_1QP = \angle OPB$
 - 2. $\angle BOP = \angle BQA$
- similar triangles (2)
- ratio segments (8)

The bottom status bar shows "Thm F D A M" and "Move".

A solution of BAMO 1999/2 with JGEX

The screenshot displays the JGEX software interface. The main workspace shows a geometric construction with the following elements:

- Horizontal line X_1X_2 with points Q and O on it.
- Vertical line segment BO perpendicular to X_1X_2 at O .
- A circle centered at M on BO , passing through A on BO and B .
- A line through Q and A intersecting the circle at P .
- Angles $\angle BQP$ and $\angle BOP$ are both labeled 12.7° .

The left sidebar shows the following list of objects and constraints:

- Fixpoint
- lines (3)
- perpendicular lines (3)
- midpoints (1)
- circles (2)
- congruent segments (1)
- congruent angles (4)
 - 1. $.[BAP] = .[APM] = .[X_1X_2BP]$
 - 1. $.[OBP] = .[BPM] = .[X_1QA]$
 - 2. $.[X_1QB] = .[OPB]$
 - 2. $.[BOP] = .[BQA]$ (highlighted)
- similar triangles (2)
- ratio segments (8)

The bottom status bar shows "Thm F D A M" and "Move".

A solution of BAMO 1999/2 with JGEX

The screenshot displays the JGEX software interface. The main workspace shows a geometric diagram with a horizontal line containing points X_1 , Q , O , and X_2 . A circle is centered at O and passes through point A . A vertical line segment OB is drawn, with M as its midpoint. A line segment AP is drawn, and a circle is constructed passing through A , M , and P . Two angles are marked as 12.7° : $\angle BQP$ and $\angle BOP$.

The left sidebar shows the following constraints:

- 1. $\angle BOP = \angle BQA$ (r13)
- 2. $\text{cyclic}(B,P,O,Q)$
- 3. $OB \perp OQ$
- 4. $PB \perp PQ$
- 3. $OB \perp OQ$
 $BO \perp X_1X_2$ (by HYP)
- 4. $PB \perp PQ$
- 5. $BP \perp AP$
- 5. $BP \perp AP$ (r10)
- 6. $\text{cyclic}(M,B,A,P)$
 M, B, A are collinear (by HYP)
- 6. $\text{cyclic}(M,B,A,P)$
 $MA = MP$ (by HYP)
 $\text{midpt}(M,AB)$ (by HYP)

At the bottom left, there is a "Move" button and a status bar with "Thm F A M Fix".

A solution of BAMO 1999/2 with JGEX

Steps of the proof

The screenshot displays the JGEX geometry software interface. On the left, a list of steps is shown:

1. $\perp(BOP) = \perp(BQA)$ (P13)
2. $\text{cyclic}(B,P,O,Q)$
3. $OB \perp OQ$
4. $PB \perp PQ$
3. $OB \perp OQ$
- $BO \perp X_1X_2$ (by HYP)
4. $PB \perp PQ$
5. $BP = AP$
6. $BP = AP$ (P10)
6. $\text{cyclic}(M,P,A,P)$
 M,P,A are collinear (by HYP)
6. $\text{cyclic}(M,P,A,P)$
 $MA = MP$ (by HYP)
 $\text{midpt}(M,AB)$ (by HYP)

The main workspace shows a geometric diagram with points X_1, Q, O, X_2 on a horizontal line. Point B is above O . A circle passes through B, P, O, Q . Point M is on BO . A red triangle MPA is shown. Angles of 12.7° are marked at Q and O . A 90.0° angle is marked at O between BO and OQ . The software interface includes a toolbar at the top and a list of tools on the right.

A solution of BAMO 1999/2 with JGEX

Steps of the proof

The screenshot displays the JGEX software interface for a geometric proof. The main workspace shows a diagram with the following elements:

- A horizontal line with points X_1 , A , O , and X_2 .
- A vertical line segment BO perpendicular to X_1X_2 at O .
- A circle tangent to X_1X_2 at O and passing through B .
- Point M is the midpoint of BO .
- A circle tangent to BO at M and passing through A and P .
- A line segment AP is drawn.
- A line segment BP is drawn.
- Angles of 12.7° are marked at A and P .

The left sidebar shows the construction steps:

1. $[BOP] = -[BQA]$ (r13)
2. $\text{cyclic}(B,P,O,Q)$
3. $OB \perp OQ$
4. $MP \perp MQ$
3. $OB \perp OQ$
8. $O \perp X_1X_2$ (by HYP)
4. $MP \perp MQ$
5. $BP \perp AP$
5. $BP \perp AP$ (r10)
6. $\text{cyclic}(M,P,A,Q)$
 M,P,A are collinear (by HYP)
6. $\text{cyclic}(M,Q,A,P)$
 $MA = MP$ (by HYP)
 $\text{midp}(M,AB)$ (by HYP)

The bottom of the interface shows the text "Thm F A M Fix" and a "Move" button.

A solution of BAMO 1999/2 with JGEX

Steps of the proof

The screenshot displays the JGEX software interface for a geometric proof. The main workspace shows a diagram with points X_1, Q, O, X_2 on a horizontal line. A vertical segment BO is drawn from O to point B . A circle is centered at O and passes through B . Point A is on BO , and M is on AB . Point P is on the circle centered at O . A dashed red circle is centered at Q and passes through B and P . A solid blue circle is centered at O and passes through B and P . Angles of 12.7° are marked at Q and O . A 90.0° angle is marked at O between BO and OQ .

The left sidebar shows the following steps in the proof:

1. $\perp(BOP) = \perp(BQA)$ (r13)
2. **cyclic(B,P,O,Q)**
3. $OB \perp OQ$
4. $PB \perp PQ$
5. $3.OB \perp OQ$
6. $BO \perp X_1X_2$ (by HYP)
7. $4.PB \perp PQ$
8. $5.BP \perp AP$
9. $6.cyclic(M,P,A,Q)$
10. MA, A are collinear (by HYP)
11. $6.cyclic(M,Q,A,P)$
12. $MA = MP$ (by HYP)
13. $midp(M,AB)$ (by HYP)

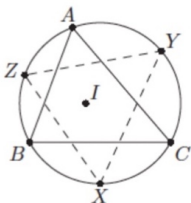
The bottom of the interface shows the text "Thm F A M Fix" and a "Move" button.

Geometry problems to apply the inscribed angle theorem

A collection by Ralf Roupec

- 6) Let ABC be an acute triangle with circumcircle k . Let X be the midpoint of the arc BC such that A is not included in it. The points Y and Z are defined analogously.

Show that the orthocenter of XYZ is the center of the incircle of ABC .



An attempt to solve Problem 6 with JGEX

The screenshot displays the Geometry Expert (JGEX) software interface. The main workspace shows a geometric construction involving a large circle with points A, B, C, X, Y, Z on its circumference. A horizontal chord BC is shown, with its midpoint F . A vertical line AX is drawn from point A to the bottom of the circle, with its midpoint O . A line YZ is drawn from point Z to point Y , with its midpoint P . A smaller circle is constructed, passing through points F, O, P . The center of this smaller circle is labeled U . The software interface includes a menu bar (File, Examples, Construct, Constraint, Action, Prove, Lemmas, Option, Help), a toolbar with various geometric tools, and a left-hand panel listing the current construction steps and constraints. The bottom status bar shows the time as 0.001 seconds and the date as 02.06.2023.

Geometry Expert

File Examples Construct Constraint Action Prove Lemmas Option Help

Fixpoint

- lines (2)
 - B, C are collinear
 - A, C, F are collinear
- perpendicular lines (3)
 - $\text{perp}[Z, A, B]$
 - $\text{perp}[Y, A, C, F]$
 - $\text{perp}[X, B, C]$
- midpoints (2)
 - midpoints[BC]
 - midpoints[F, CA]
- circles (2)
 - circle[$U, ABCZY$]
 - circle[F]
- congruent segments (1)
 - $AU = UZ = BU = CU = UX = UY$

Time: 0.001 Seconds

Suchen 15:28 02.06.2023

Conclusion on JGEX

- + Sophisticated user interface
- Too complex for beginners
- Only English, Chinese, German, Portuguese, Persian and Serbian are supported (some of them just partially)
- + Visual proofs are very useful even for university students
- Formulating the problem can be challenging

- More problems to checks
- Find suggestions to improve the user interface (possibly joint work with Alexander Vujic)
- Measure impact of JGEX on structured thinking at secondary level
- Advertise JGEX among teachers (with workshops and tutorials)

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