
ADG
2023, 20th September, Belgrade, Serbia
Dobro došli! Welcome!

SOLVING SOME GEOMETRY PROBLEMS OF THE NÁBOJ 2023 CONTEST WITH AUTOMATED DEDUCTION IN GEOGEBRA DISCOVERY

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- The Náboj contest
- Mathematical background
- Problems that can be solved with GeoGebra Discovery
- Problems that require further improvements
- Conclusion

Abstract

- solving geometry problems of the Náboj 2023 competition with GeoGebra Discovery
- analyzing the difficulty of feeding the problem into the machine
- set further goals to make the problems of this type of contest even more tractable in future

Introduction

- trying to solve competitive problems (from Náboj 2023 contest) with an ADG algorithm in the background
- using the GeoGebra fork GeoGebra Discovery
  - measurements symbolically
  - full proof in the background created – no user information
- electronic aids are restricted

The Náboj contest

- international mathematical competition
- teams of five high-school students (representing their school)
- 120 minutes → team with most correctly solved problems wins (one solved – get new one)
- require a certain amount of inventiveness and ingenuity
- final result: fraction, non-trivial algebraic number or root expression

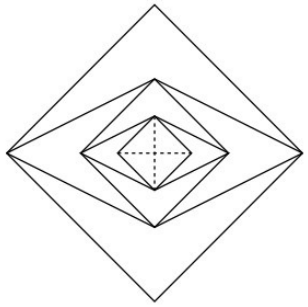
Mathematical background

- GeoGebra Discovery (GGD) uses the Recio-Vélez method
- some problems can be simplified
- GGD in some cases takes decisions itself e.g. to substitute A = (0, 0)
- the solving process by using GGD can get more complicated compared to the official solution

Mathematical background: how the program solves Problem 6 from Náboj 2023

Problem setting and official solution:

Problem 6. The rhombus flower grows according to the following pattern: In the middle there is a square blossom with two diagonals of length 1. In the first step the horizontal diagonal is doubled creating a new quadrilateral. In the next step the vertical diagonal is doubled and again a new quadrilateral blossom is generated. This procedure is continued until there is a flower with five quadrilateral blossoms. Find the perimeter of the outer (i.e. the fifth) blossom.

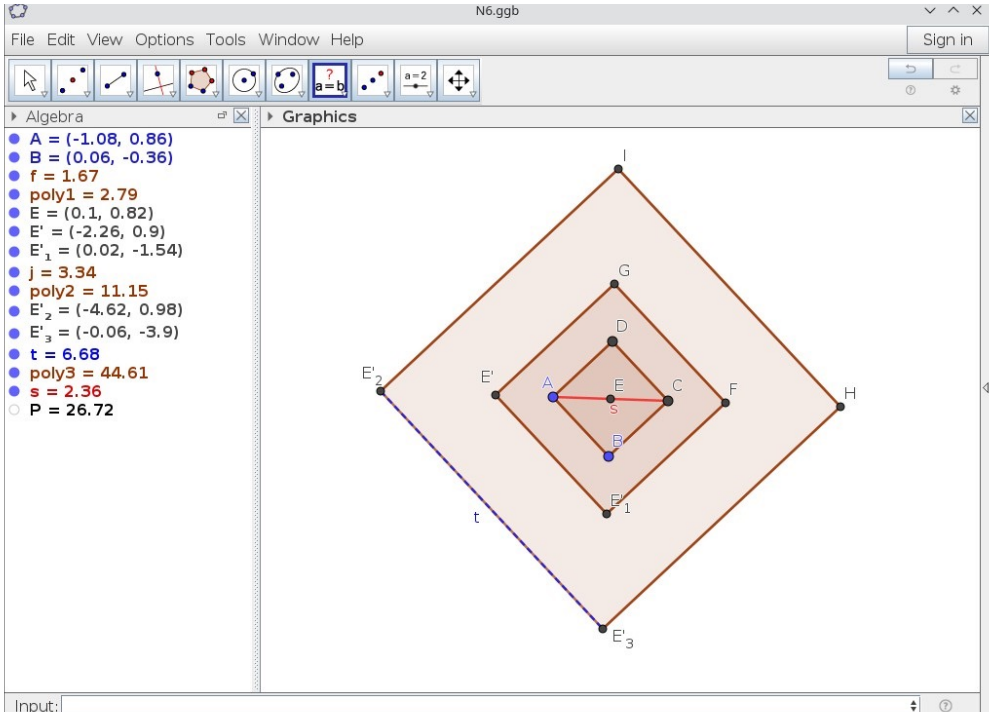


Result. $8\sqrt{2}$

Solution. The fifth blossom is a square with diagonals of length 4, hence the length of its side is $2\sqrt{2}$ and the perimeter equals $8\sqrt{2}$.

Mathematical background: how the program solves Problem 6 from Náboj 2023

Sketch in GeoGebra Discovery



Protocol

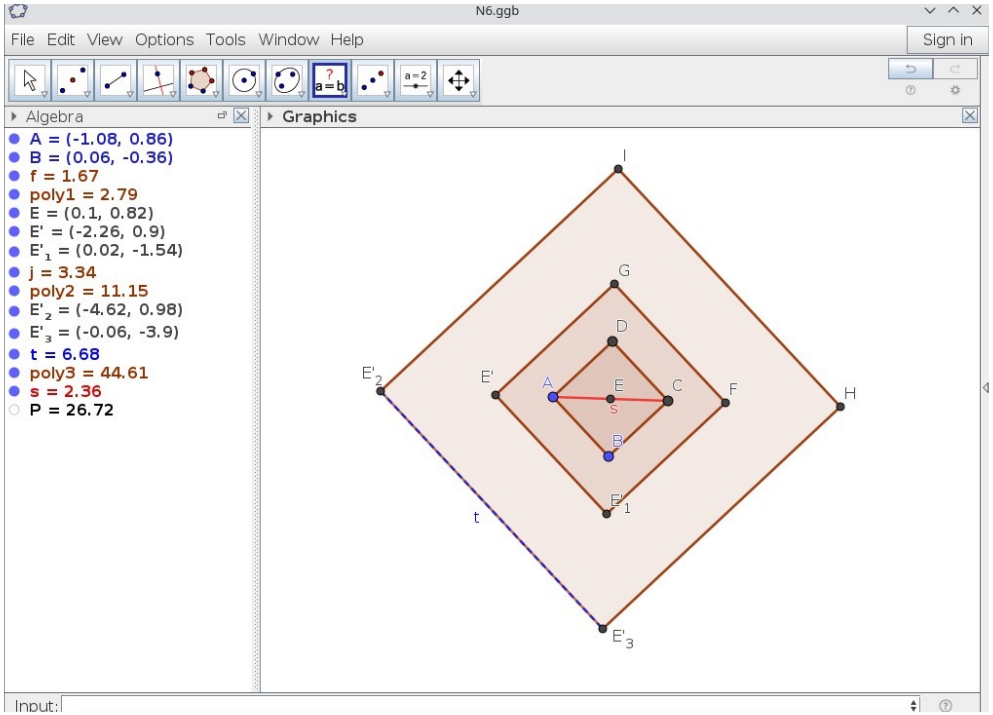
Table 1: Construction protocol in GeoGebra Discovery for Problem 6

No.	Name	Toolbar Icon	Description
1	Polygon poly1		Polygon(A, B, 4)
2	Point E		Midpoint of A, C
3	Point E'		E mirrored at A
4	Point E'_1		E mirrored at B
5	Polygon poly2		Polygon(E', E'_1, 4)
6	Point E'_2		E mirrored at E'
7	Point E'_3		E mirrored at E'_1
8	Polygon poly3		Polygon(E'_2, E'_3, 4)
9	Segment t		Segment E'_2, E'_3
10	Segment s		Segment A, C
11	Number P		4t

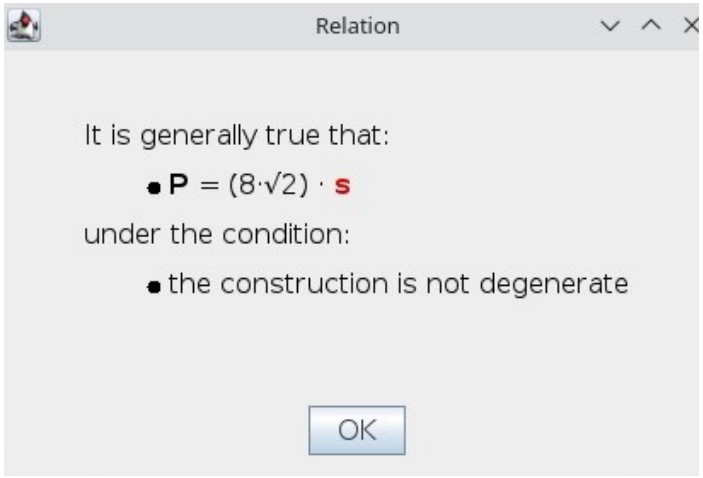
https://autgeo.online/

Mathematical background: how the program solves Problem 6 from Náboj 2023

Sketch in GeoGebra Discovery

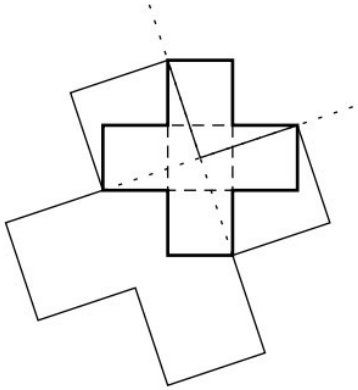


Relation Report:



Problems that can be solved with GeoGebra Discovery

Problem 15: Pentominos



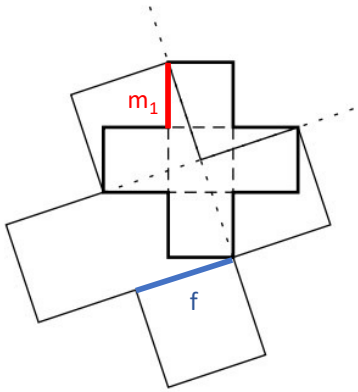
Antonia drew a small X-pentomino made of 5 congruent squares. Then she drew two perpendicular diagonals of this pentomino with dotted lines. Finally she constructed a bigger X-pentomino with some of the sides lying on the diagonals of the small pentomino as in the figure. Find the ratio of the area of Antonia's big pentomino to the area of the small one.

Problem 47: A triangle and a circle

Let O be the circumcenter of triangle ABC. Let further points D and E lie on the segments AB and AC, respectively, so that O is the midpoint of DE. If $AD = 8, BD = 3$, and $AO = 7$, determine the length of CE.

Problem 15: Pentominos

Problem setting:



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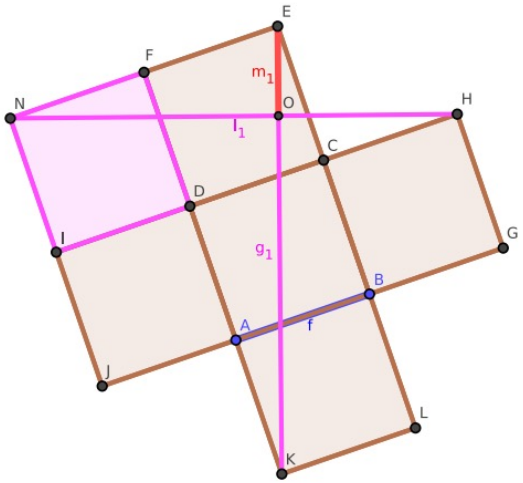
Sketch made with GeoGebra Discovery



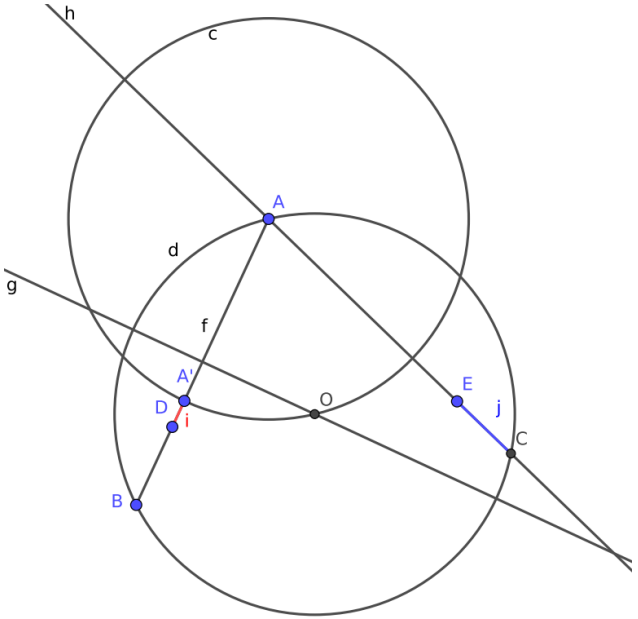
asking the relation between m_1 and f we obtain that $f = \frac{1}{2} \cdot \sqrt{10} \cdot m_1$
the ratio of the areas must be ${f^2}:{m_1}^2 = 10 : 4 = 5 : 2$

Problem 47: A triangle and a circle

Problem setting:

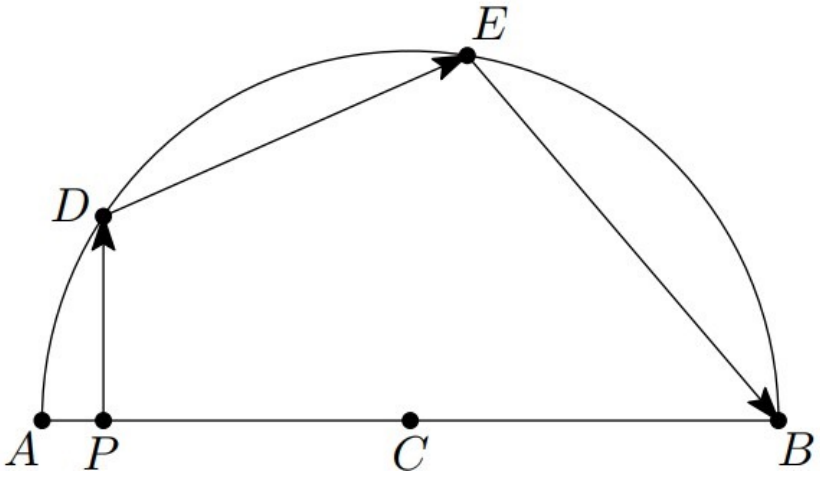
Let O be the circumcenter of triangle ABC. Let further points D and E lie on the segments AB and AC, respectively, so that O is the midpoint of DE. If $AD = 8, BD = 3$, and $AO = 7$, determine the length of CE.

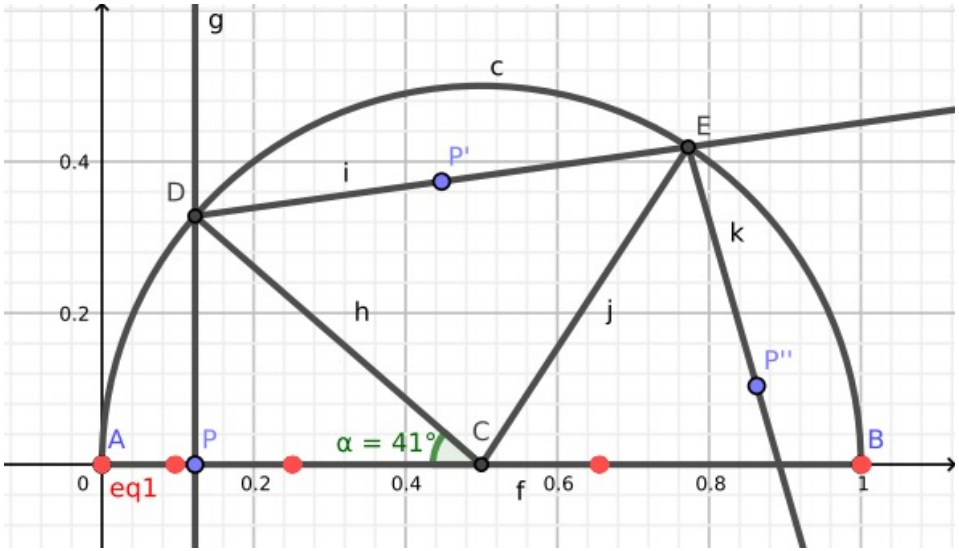
Sketch made with GeoGebra Discovery



Problems that require further improvements

Problem 25. Consider a semi-circle with centre C and diameter AB. A point P on AB satisfies the following. A laser beam leaves P in a direction perpendicular to AB, bounces off the semicircle at points D and E following the rule of reflection, that is, $\angle PDC = \angle EDC$ and $\angle DEC = \angle BEC$, and then it hits the point B. Determine $\angle DCP$ in degrees.



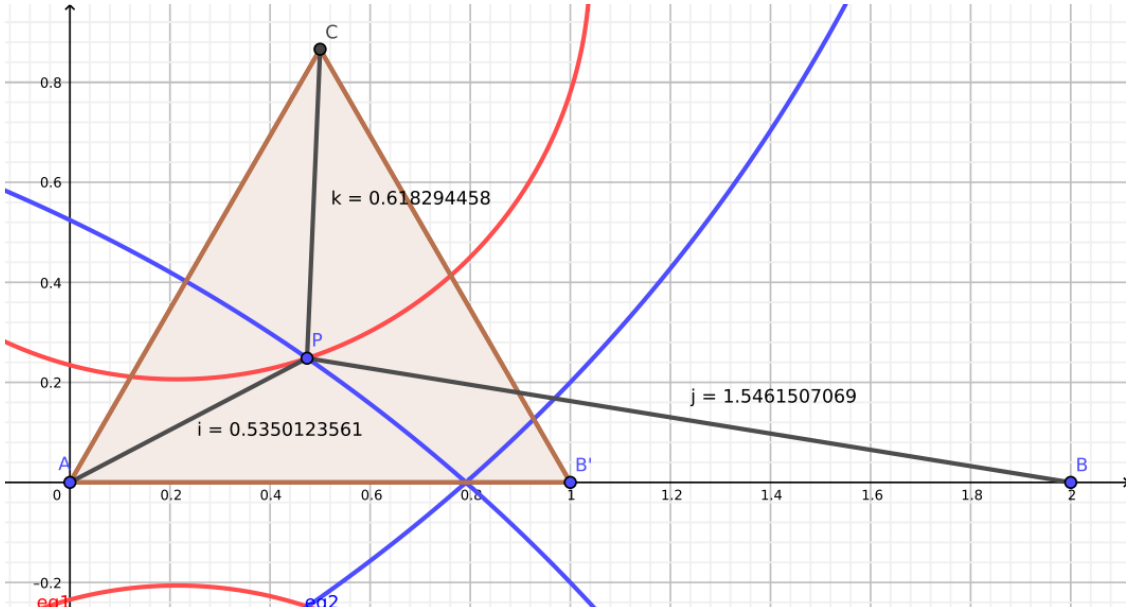


Problems that require further improvements

Problem 58. A point P is located in the interior of triangle ABC. If

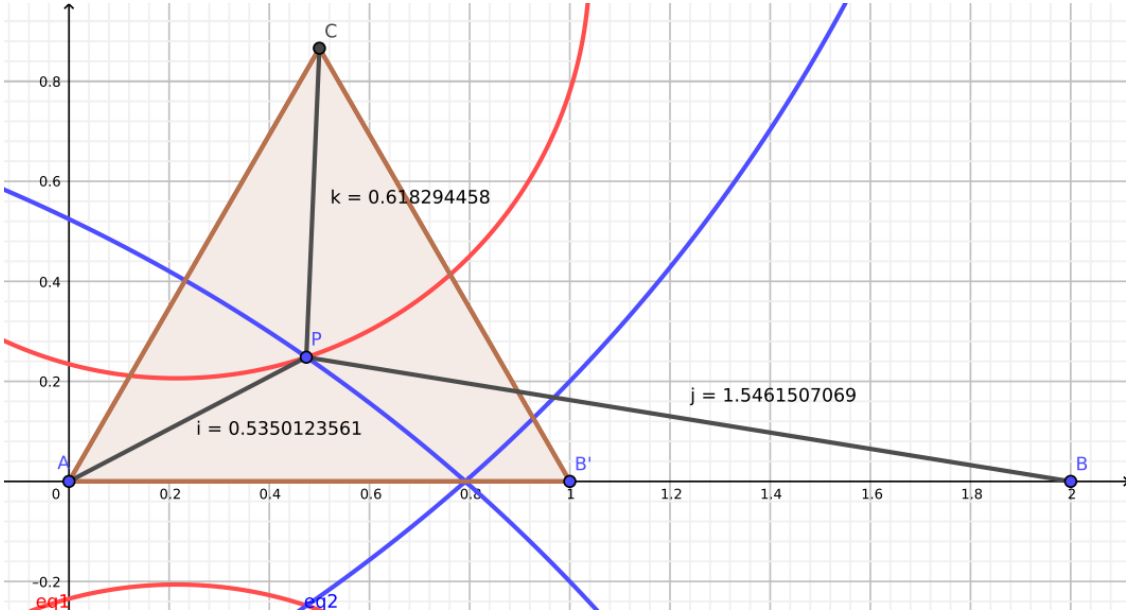
$$AP = \sqrt{3}, \quad BP = 5, \quad CP = 2, \quad AB : AC = 2 : 1, \quad \text{and} \quad \angle BAC = 60°,$$

what is the area of triangle ABC?



Problem 58:

Sketch made with GeoGebra Discovery



Conclusion

- the right steps are taken → GeoGebra Discovery can be a useful tool
- can be misleading in some situations
- mathematical background knowledge necessary
- good knowledge of the software is unavoidable
- further improvement:
  - full support of computing angles and areas
  - developments towards symbolic computations that are based on implicit assumptions

Technology will not replace great teachers but technology in the hands of great teachers can be transformational. **– George Couros**

AI is a tool. The choice about how it gets deployed is ours. **– Oren Etzioni**

Hvala vam!

Thank You for Your attention!