

OK Geometry - observing dynamic constructions

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You can download OK Geometry at

www.ok-geometry.com

Please download the version 19.4.4 .

Hereby shown material (and more) is in

ADG 2023 section

Unzip and launch OKGeometry_19_4.exe

OK Geometry - A tool for observing dynamic constructions

The screenshot displays the OK Geometry Plus software interface. The main workspace shows a large triangle with vertices A , B , and C . A point P is located inside the triangle. Lines connect P to each vertex, and lines connect P to the midpoints of each side, forming a smaller triangle $A'B'C'$ and three other triangles $PA'B'$, $PB'C'$, and $PC'A'$. The triangles $PA'B'$, $PB'C'$, and $PC'A'$ are shaded green, while the outer triangles $PA'B'$, $PB'C'$, and $PC'A'$ are shaded yellow. The software interface includes a menu bar (File, Configure, Commands, Help, Development), a toolbar with various construction tools, and a panel on the left showing a list of observed properties.

Observed properties:

- points (27)
- collinear points (3)
- trapeziums (18)
- equilateral triangles (3)
- isosceles triangles (12)
- congruent triangles (11)
- similar triangles (19)
- congruent segments (4)
- congruent segments (relation) (9)
- ratio of distances (94)
- parallel lines (3)
- congruent angles (71)
- congruent angles (relation) (920)
- special ratios of angles or supplements (1)
- area of triangles (21)
- area of triangles (relation) (78)
- ratio of areas of triangle (60)
- points on a circle (4)
- tangent circles (9)
- tangent circles/lines (18)

OKG - A tool for observing dynamic constructions

3 working modes

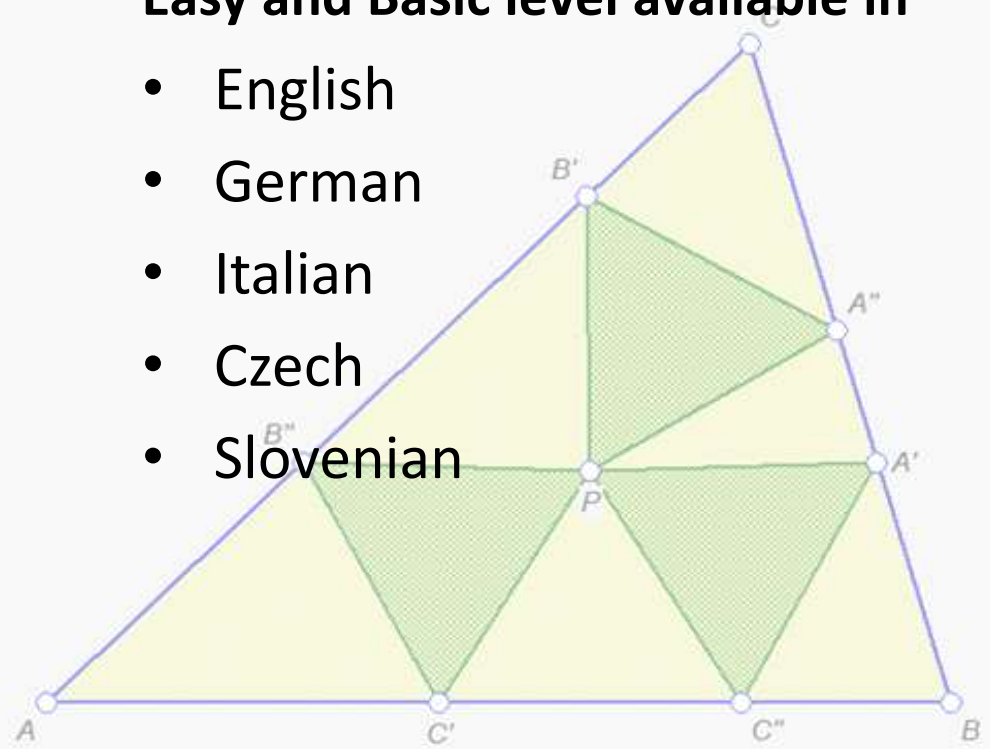
- **Easy** (lower secondary level)

- **Basic** (upper secondary level)

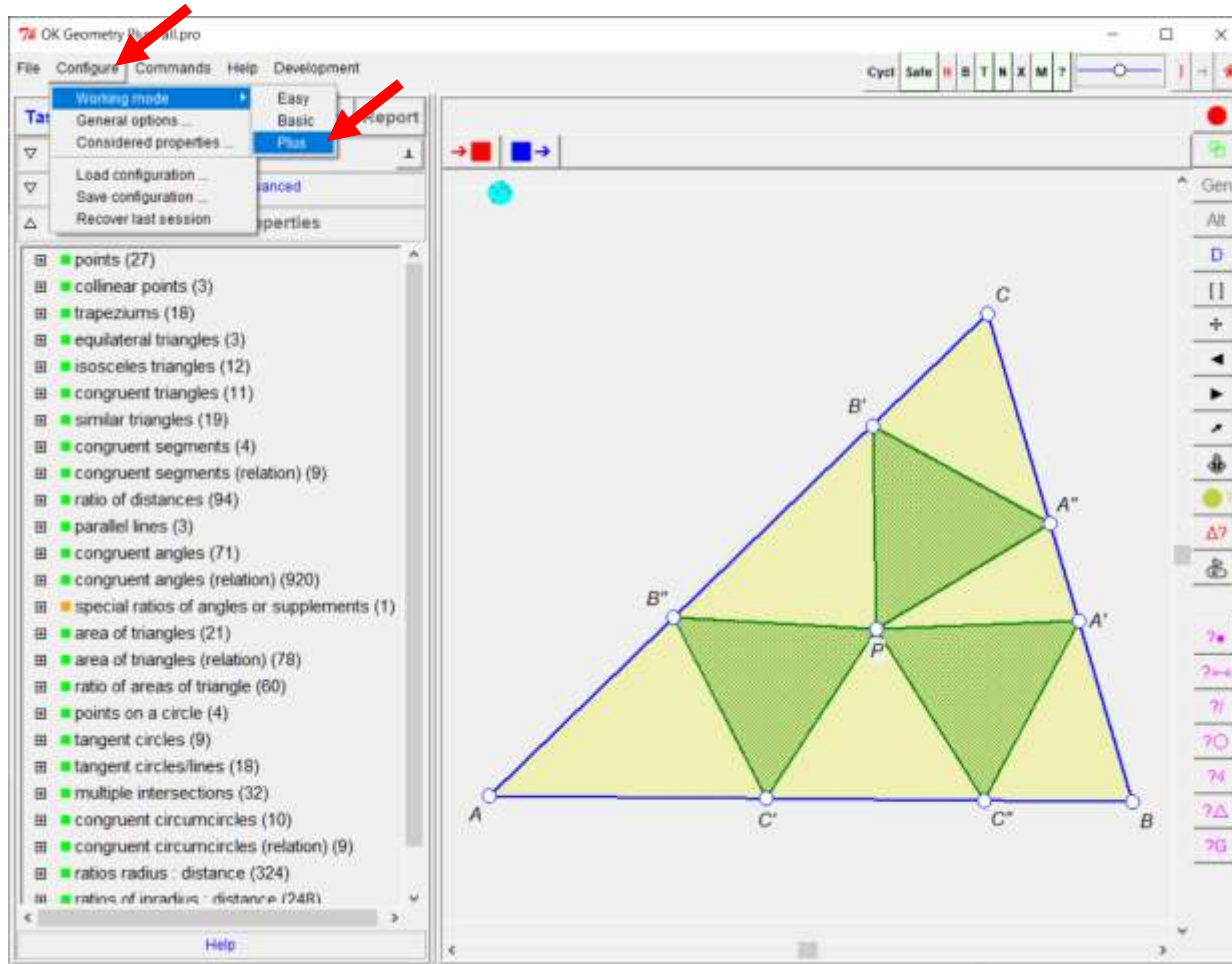
- **Plus**

Easy and Basic level available in

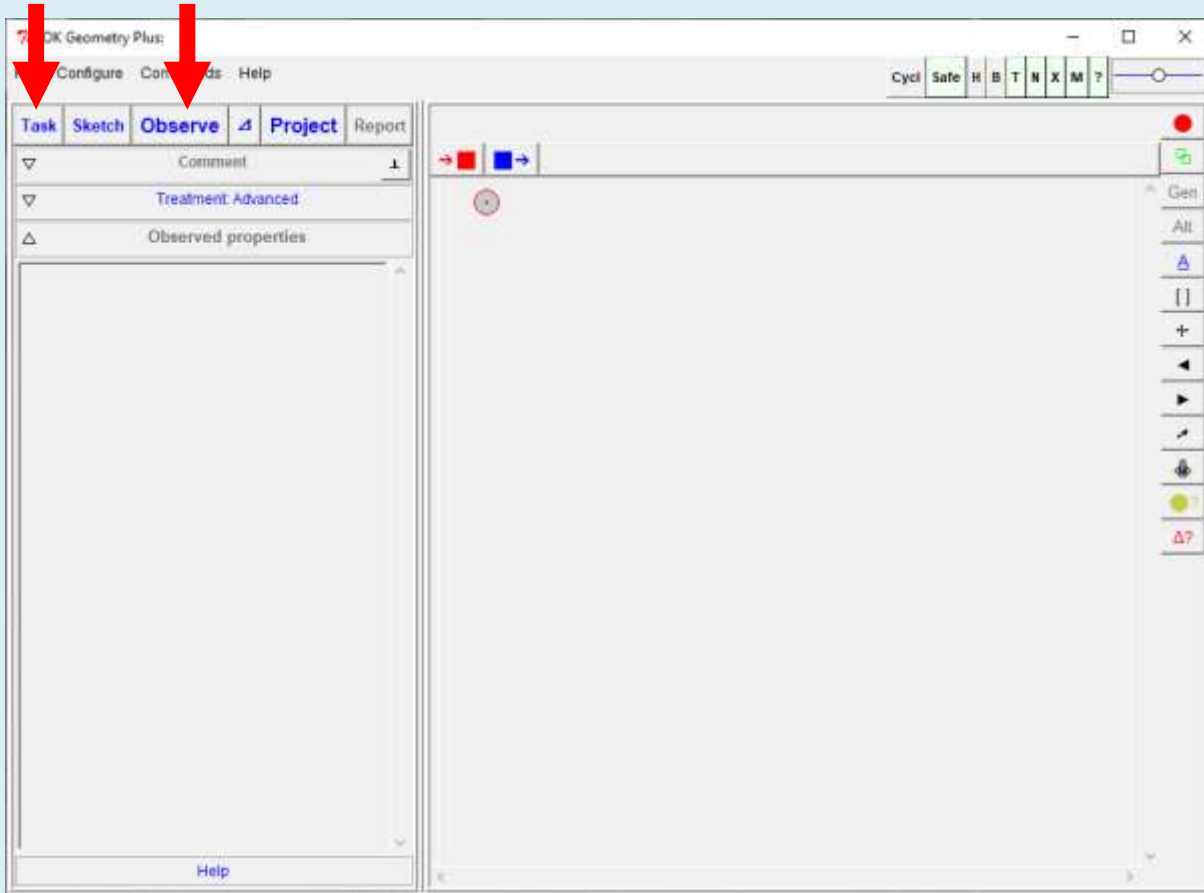
- English
- German
- Italian
- Czech
- Slovenian



OKG - A tool for observing dynamic constructions

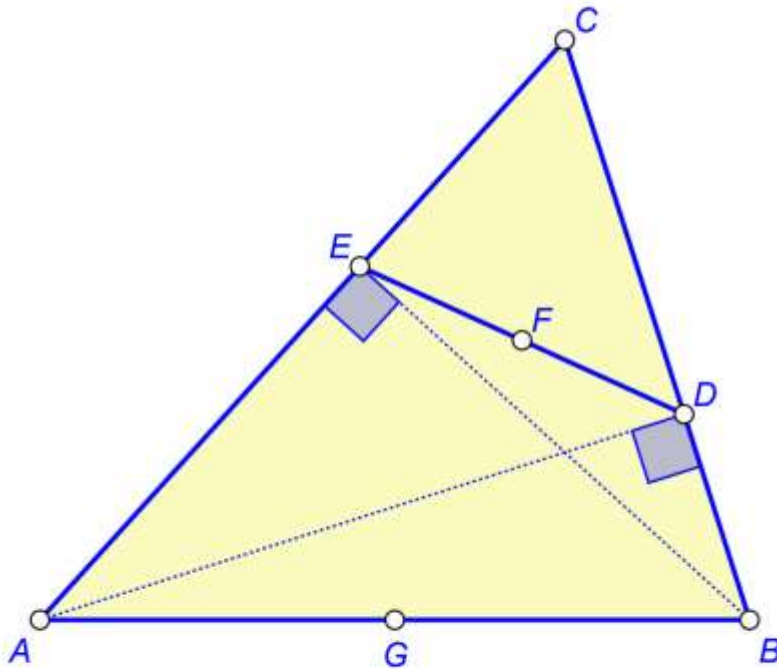


Simple observation of dynamic constructions



- Observe properties of a dynamic construction
- ‘Restricted’ observation
- Observing algebraic relations

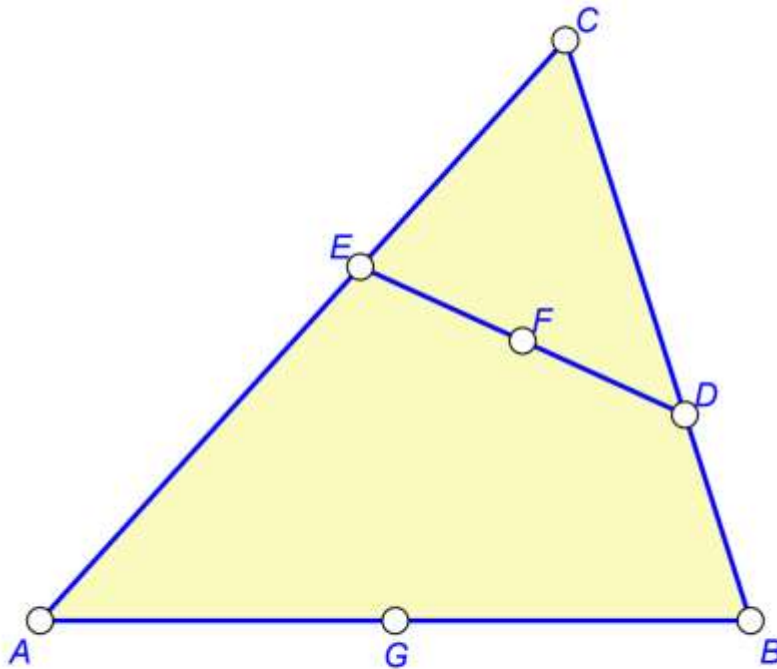
Observing imported constructions



- ABC - a triangle
- D – base of A -altitude
- E – base of B -altitude
- F – midpoint of DE
- G – midpoint of AB

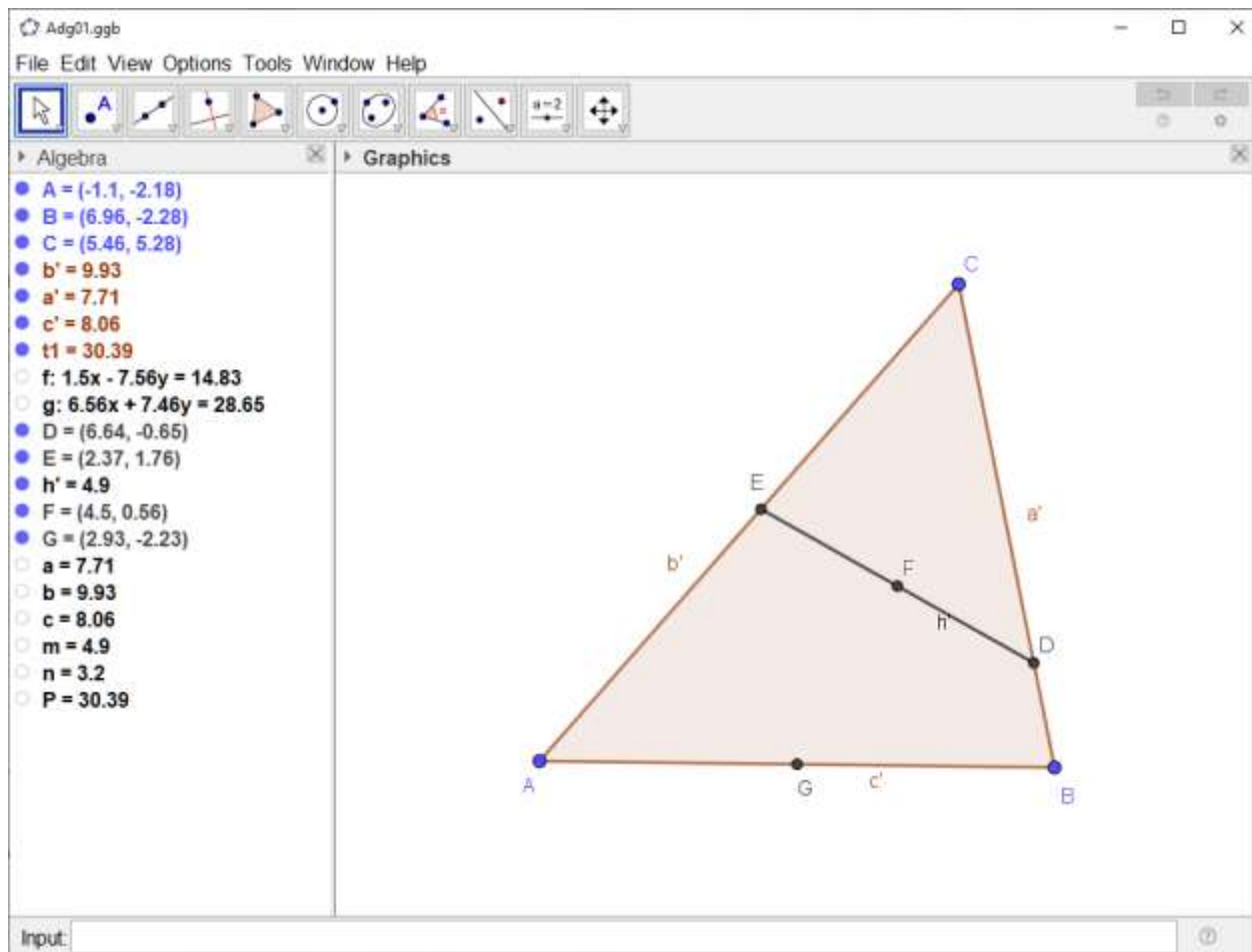
Observe the properties of this configuration.

Observing imported constructions

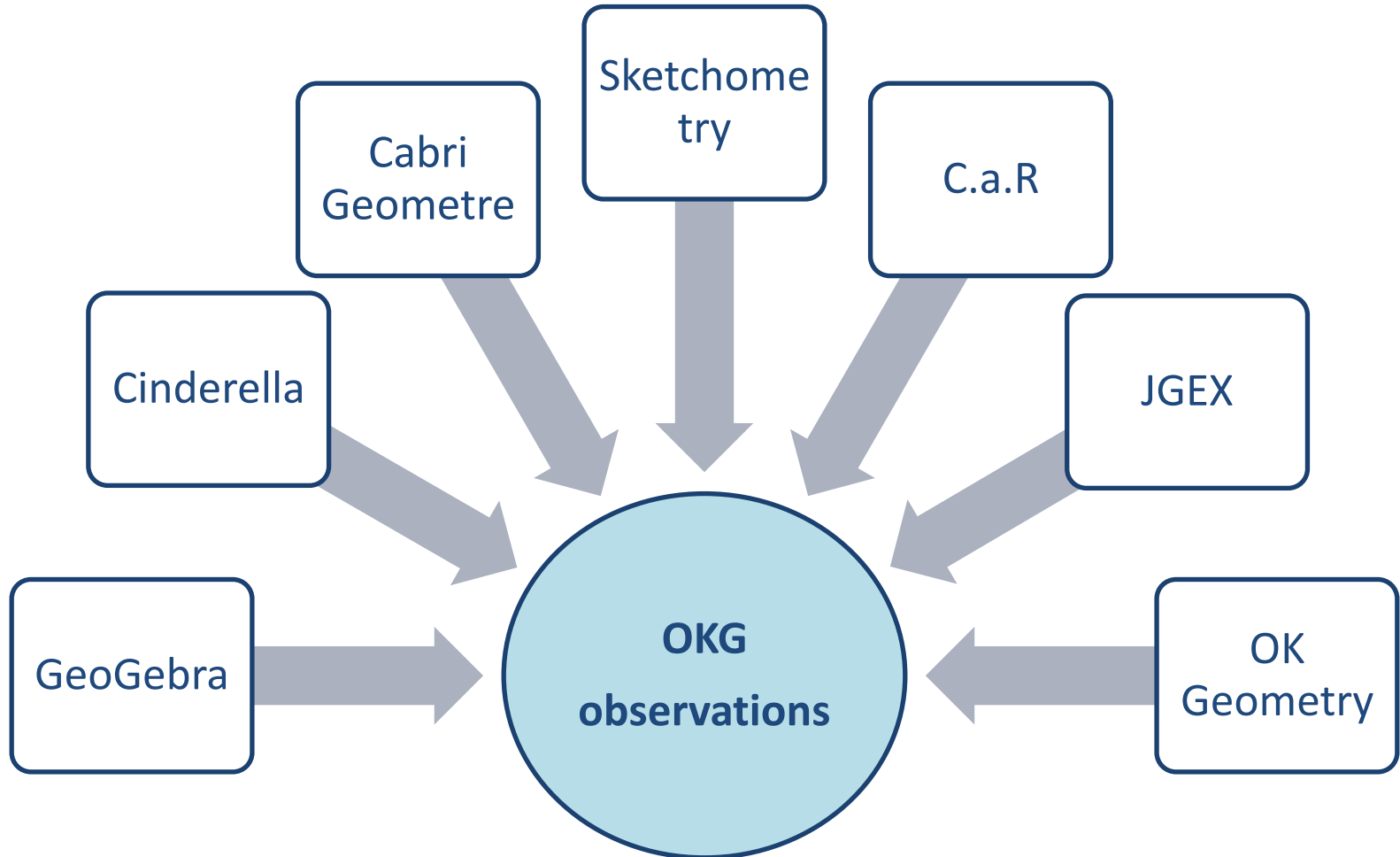


- ABC - a triangle
- D – base of A -altitude
- E – base of B -altitude
- F – midpoint of DE
- G – midpoint of AB

Observe the properties of this configuration.



Importing constructions from DGS



Observing imported constructions

The screenshot displays the OK Geometry Plus software interface. The main workspace shows a large triangle ABC with a point F inside. Lines connect F to the vertices A , B , and C . A red shaded region is formed by A , C , and F . A green shaded region is formed by C , F , and D . A yellow shaded region is formed by F , D , and B . Points E and G are also marked on the diagram.

The left sidebar contains the 'Observed properties' panel, which lists various geometric relationships. A red arrow points to the 'similar triangles' category, which includes the entry 'ACG,CDF'. Other categories include points, collinear points, isosceles triangles, right triangles, congruent triangles, congruent segments, ratio of distances, sum of lengths, congruent angles, special ratios, area of triangles, and ratio of areas.

The top menu bar includes 'File', 'Configure', 'Commands', 'Help', and 'Development'. The 'Observe' tab is selected. The status bar at the top right shows 'similar triangles' and 'ACG,CDF'.

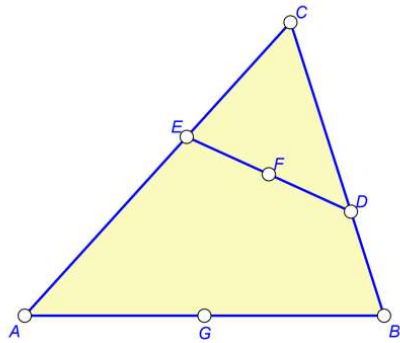
Observing imported constructions

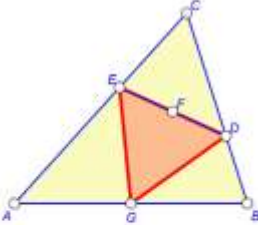
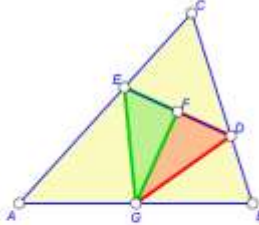
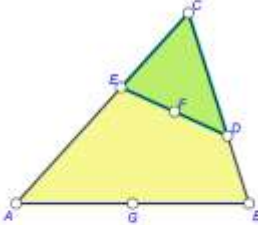
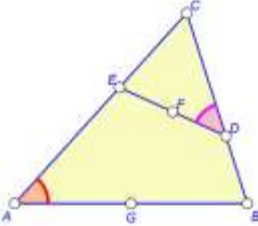
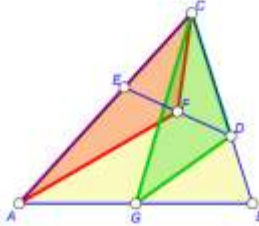
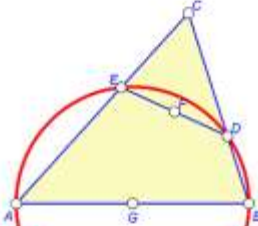
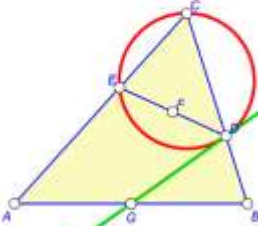
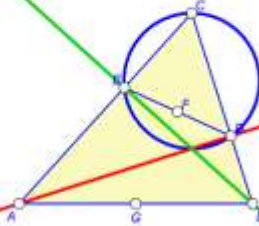
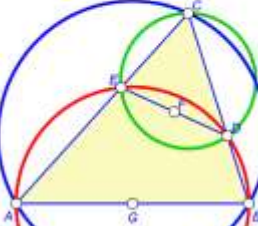
The screenshot displays the OK Geometry Plus software interface. The main workspace shows a yellow-shaded triangle with vertices A, B, and C. A line segment EF is drawn, with E on AC and F on BC. Point D is on BC, and G is on AB. A red arrow points to the text "congruent angle" above the construction. The toolbar at the top includes icons for "File", "Configure", "Commands", "Help", and "Development". The left sidebar contains tabs for "Task", "Sketch", "Observe", "Project", and "Report". The "Observe" tab is active, showing a list of observed properties for the construction (BE#DE). The list includes:

- congruent angles (1)
 - ABG#AD-ACE#FG-AD#DG-BE#DEF
- special ratios of angles or supplements (1)
 - ABG#ACE-ACE#EG-AD#FG-BCD#DE

Below the list, it states "2 properties not examined". A red arrow points to the "Advanced" button in the "Level of observation analysis" section. The right sidebar shows a list of icons for various geometric objects, including a point (?*), a line (?-), a circle (?), a triangle (?△), and a quadrilateral (?□).

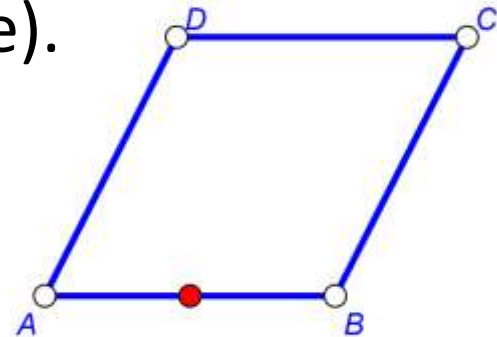
Observing imported constructions



 <p>2 <i>Isosceles triangle GDE</i></p>	 <p>3 <i>Congruent right triangles</i></p>	 <p>4 <i>Similar triangles</i></p>
 <p>5 <i>Congruent angles</i></p>	 <p>6 <i>Same area</i></p>	 <p>7 <i>Points on a circle</i></p>
 <p>8 <i>Tangent line</i></p>	 <p>9 <i>Multiple intersections</i></p>	 <p>10 $r_1^2 + r_2^2 = r_3^2$</p>

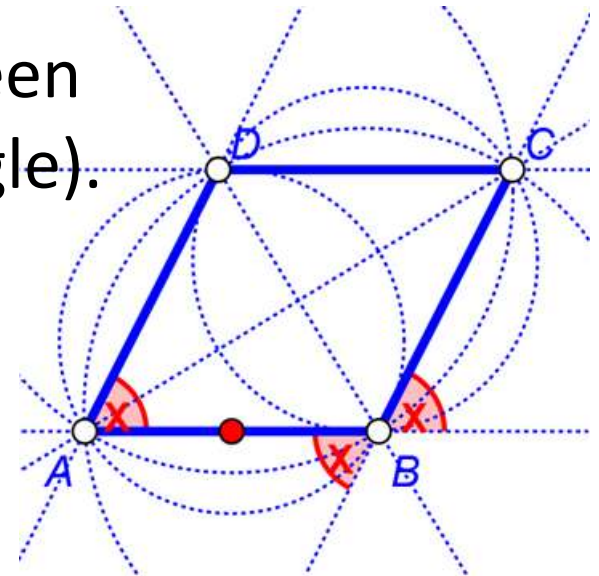
Understanding properties

- OKG considers the displayed objects and objects passing through **labelled points**.
- Advice: label 3-12 relevant points.
- OKG considers only angles between lines (angle \equiv supplementary angle).
- OKG ignores trivial congruences of angles between lines.



Understanding properties

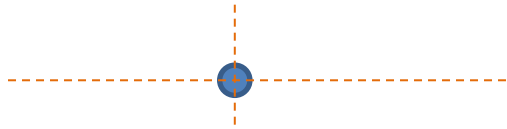
- OKG considers the displayed objects and objects passing through **labelled points**.
- Advice: label 3-12 relevant points.
- OKG considers only angles between lines (angle \equiv supplementary angle).
- OKG ignores trivial congruences of angles between lines.



Models of geometry

Static model

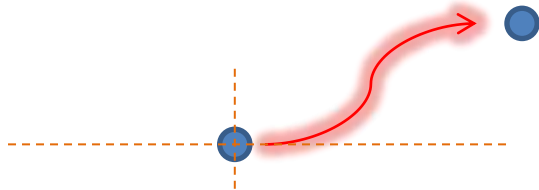
Free point $A(3,5)$



Dynamic model

Free point

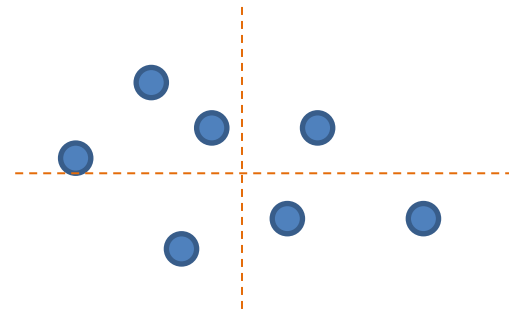
$A(3,5) \rightarrow A(x,y)$



Stochastic dynamic model

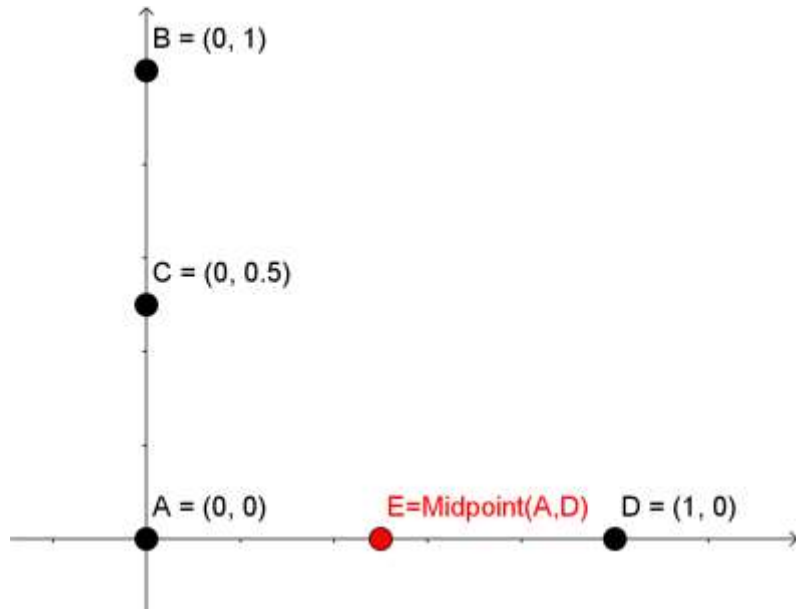
Free point $A(3,5) \rightarrow$

$\{(x_1, y_1), (x_2, y_2), \dots, (x_k, y_k)\}$

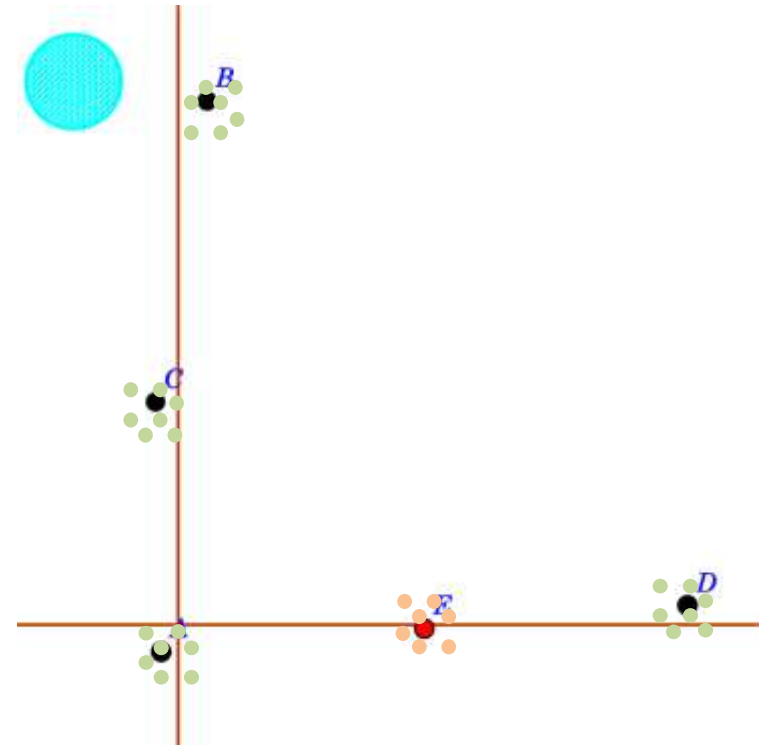


Randomisation of constructions

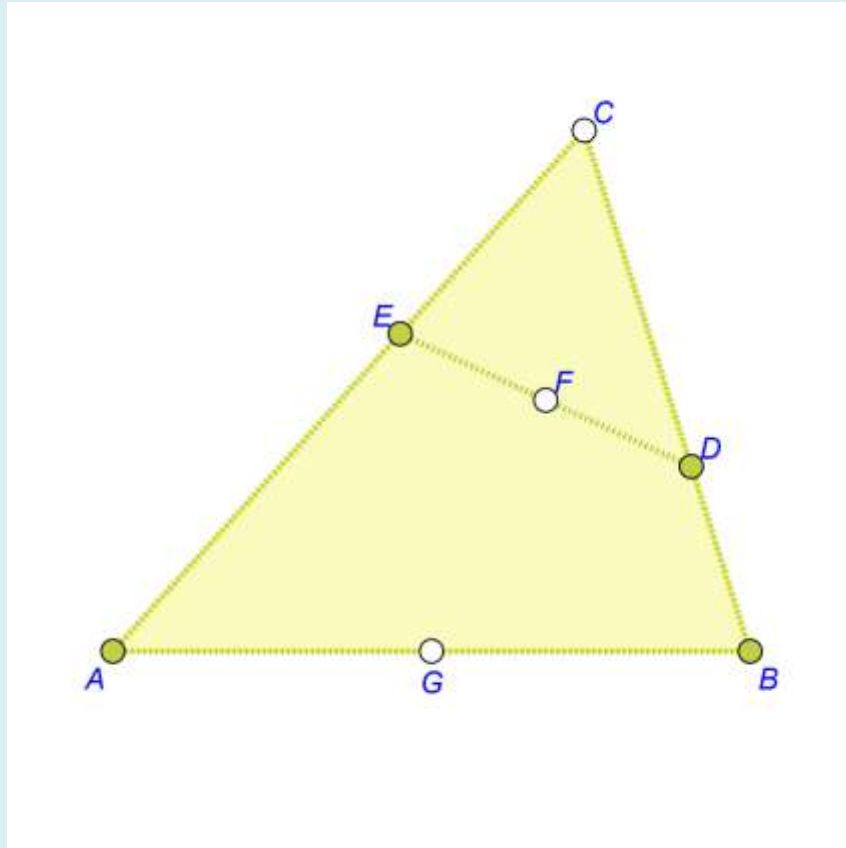
GeoGebra



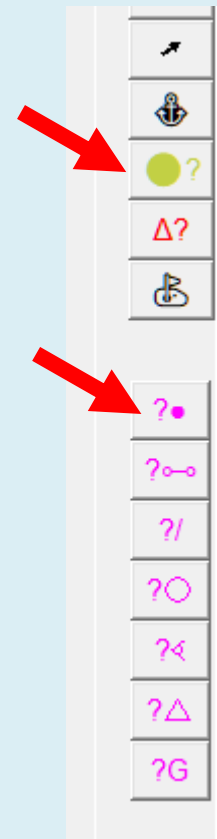
Instance of construction



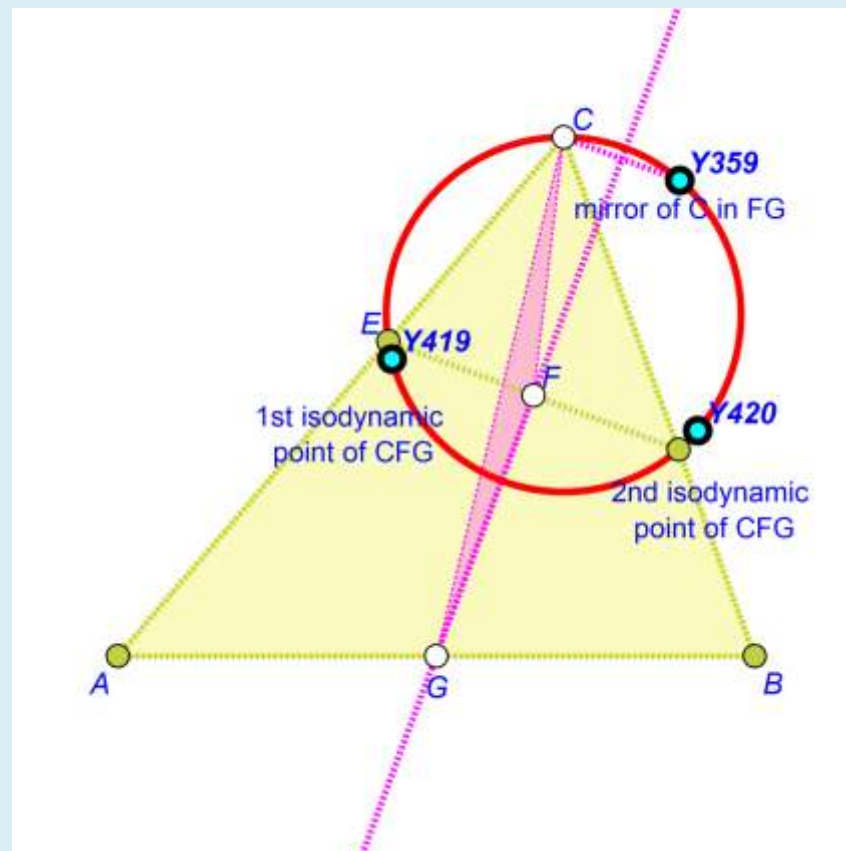
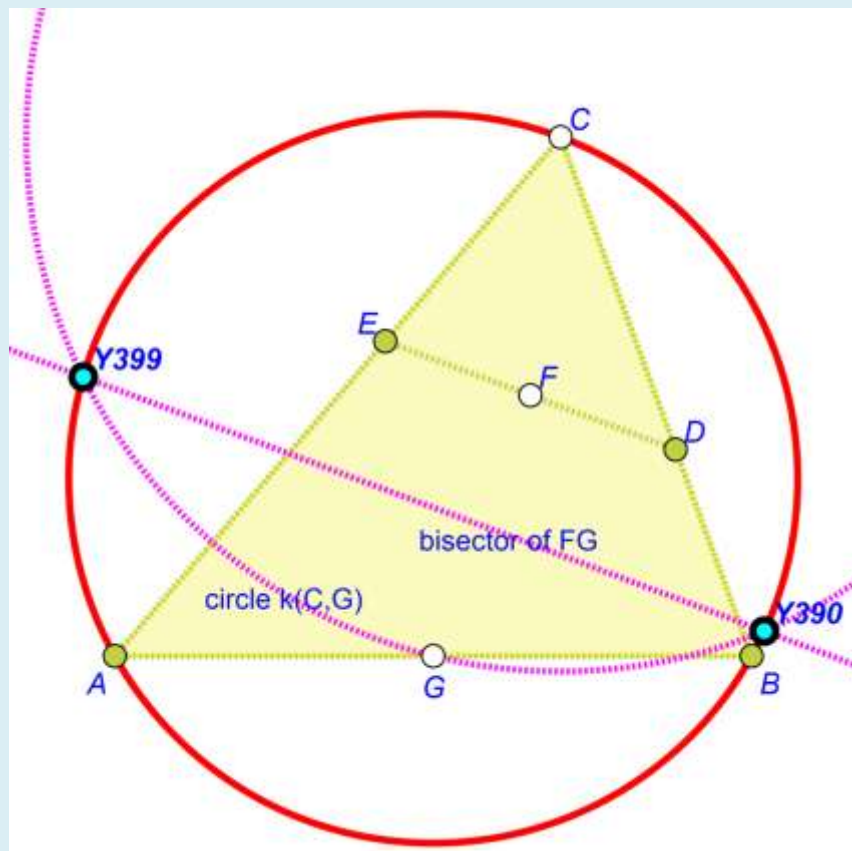
Advanced observation

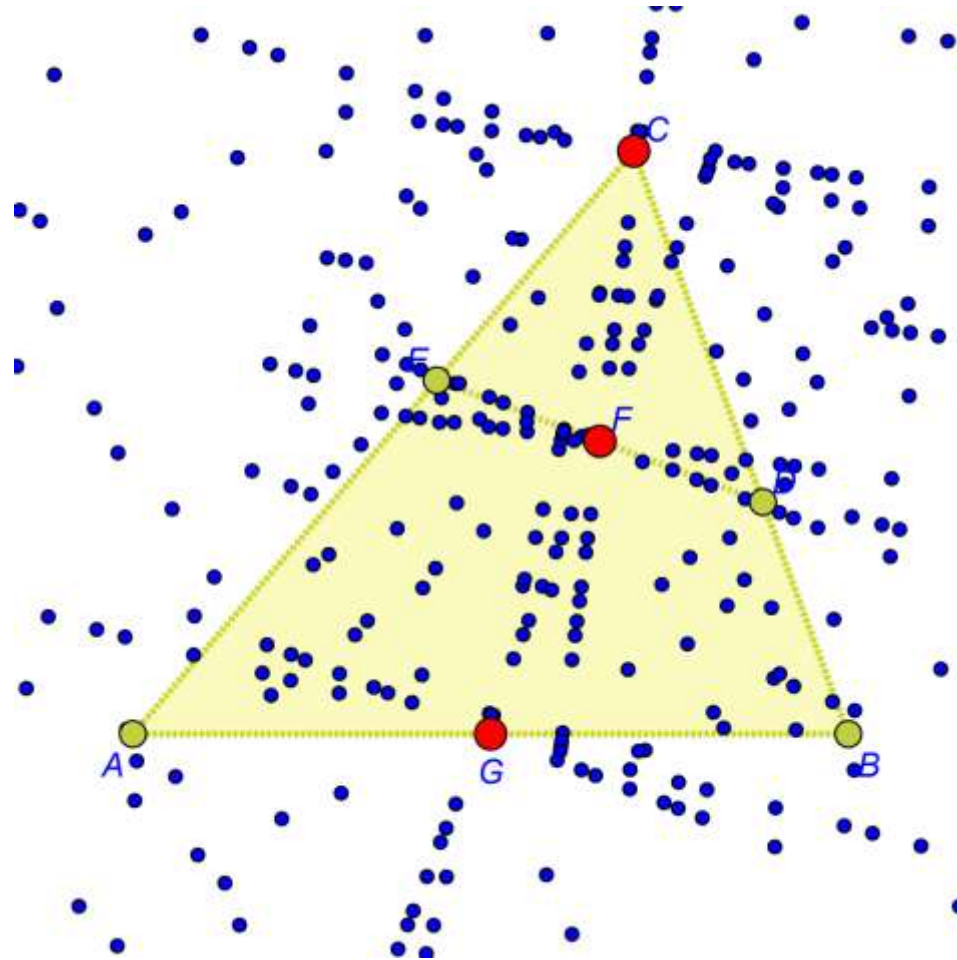


- How to **construct** the triangle ABC from known positions of points C, F, G.

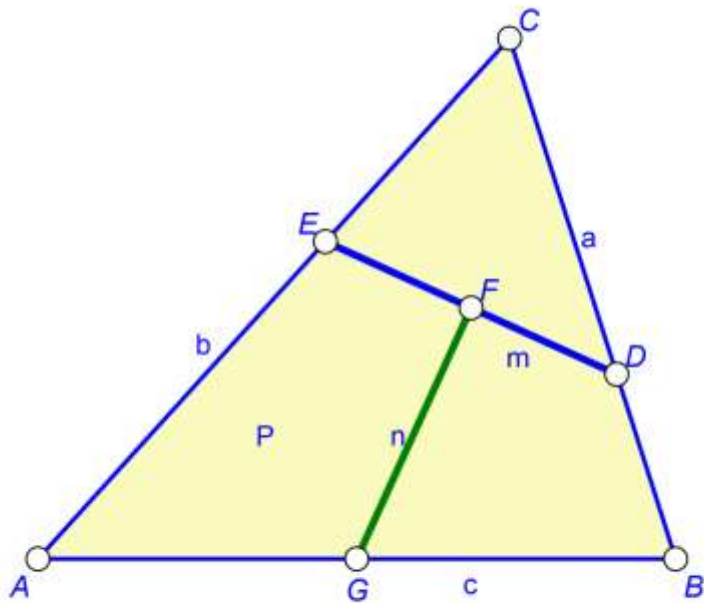


Advanced observation





Observing algebraic relations



$$P = \text{Area}(A,B,C)$$

$$a = \text{Distance}(B,C)$$

$$b = \text{Distance}(C,A)$$

$$c = \text{Distance}(A,B)$$

$$m = \text{Distance}(D,E)$$

$$n = \text{Distance}(F,G)$$

Note. Use explicit measurements.

Observing algebraic relations

The image shows a software interface with a menu bar (File, Configure, Commands, Help) and a 'Commands' menu. A red arrow points to the 'Commands' menu, and another red arrow points to the 'Observe formulae' option. A third red arrow points to the 'Simple' sub-option. To the right, a dialog box titled 'Observe formulae' is open, showing a list of variables (P, a, b, c, m, n) with 'a', 'b', 'c', and 'm' selected. The dialog has a 'Mode' tab set to 'Simple - observe relations between measured quantities'. It includes a 'Transform variables' checkbox (checked), a 'Notation' dropdown set to 'compact', and a 'Strength' dropdown set to '2'. A warning message states 'Observed formulae can be incorrect or missed!'. Below this, it shows 'Ref. triangle = ABC; Observed formula for: a,b,c,m' and the formula $(-1/2)*a^2*c + a*b*m - 1/2*b^2*c + 1/2*c^3 = 0$. At the bottom of the dialog, there are buttons for 'Observe', 'Help', and 'Exit'. A red box highlights the formula $m = \frac{c \cdot (a^2 + b^2 - c^2)}{2 \cdot a \cdot b}$.

Observing algebraic relations

- Consider several instances of a construction to obtain several instances of parameters (x_1, x_2, \dots, x_k) .
- Solve the a system of linear equations

$$\sum_{n_1+n_2+\dots+n_k < r} \alpha_{n_1, n_2, \dots, n_k} x_1^{n_1} x_2^{n_2} \dots x_k^{n_k} = 0$$

- Technical problems...

The principle of simple observation

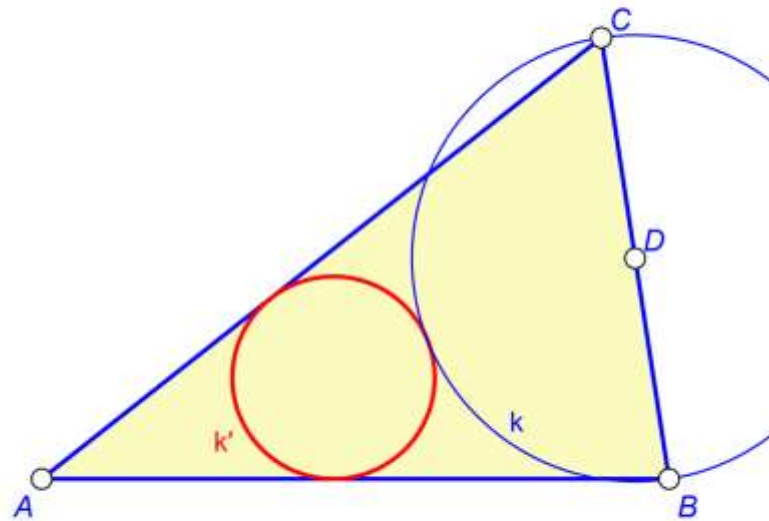
A construction

Random realisations
(rand_fig1, rand_fig2, rand_fig3,...)

Common simple numer. properties
(eg. $AB=AC$)

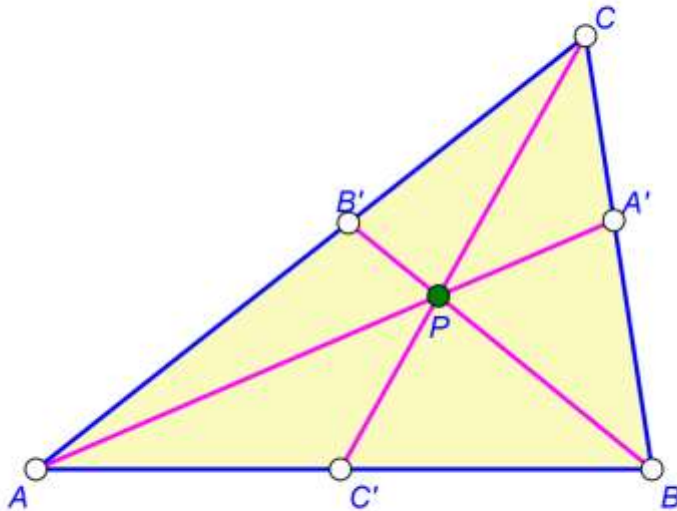
Textual elaboration for properties
(eg. ABC is isosceles)

A 'difficult' object



- ABC – an acute triangle
- $k = k(D,B)$ – circle with diameter BC
- k' – a circle inscribed in the 'triangle' bound by AB, k and CB.
- Analyse the circle k' .

An 'implicit' object



- ABC – a triangle
- P – a point
- AA', BB', CC' – Cevian lines of P in ABC.
- **AA' ≡ BB' ≡ CC'**

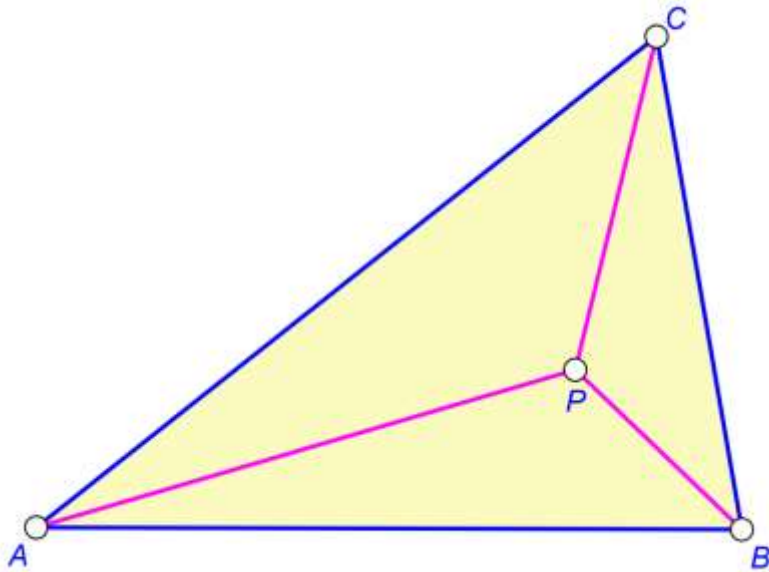
Investigate!

An optimisation problem

ABC – **reference triangle**

P – point on plane that

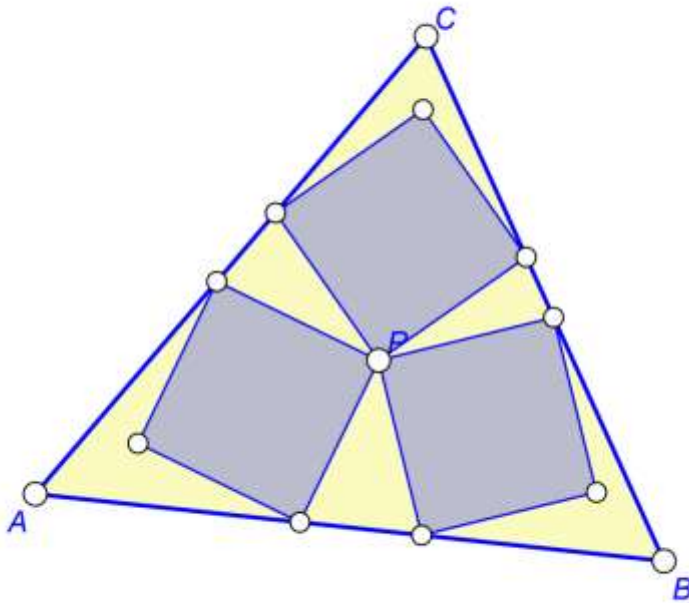
minimises
 $|AP| + |BP| + |CP|$.



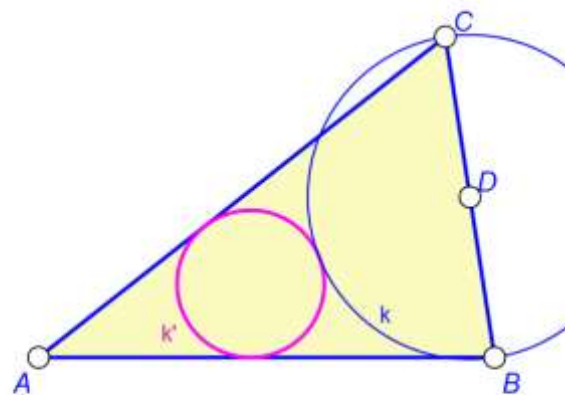
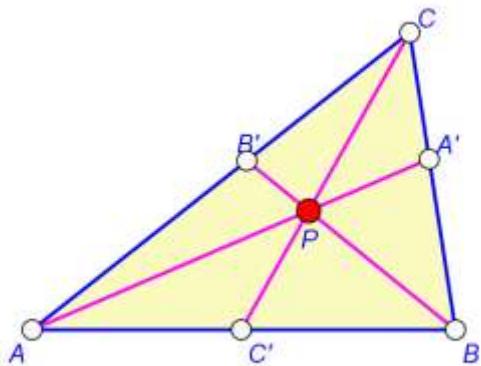
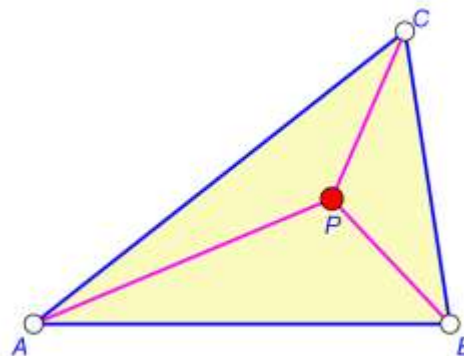
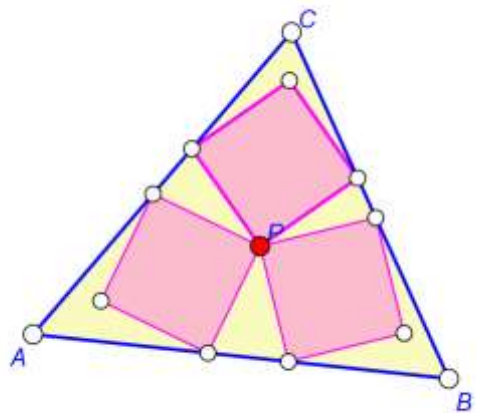
Analyse the position of
such a point P.

A nice problem

How to inscribe 3 congruent squares into a given triangle ABC as shown in the figure?



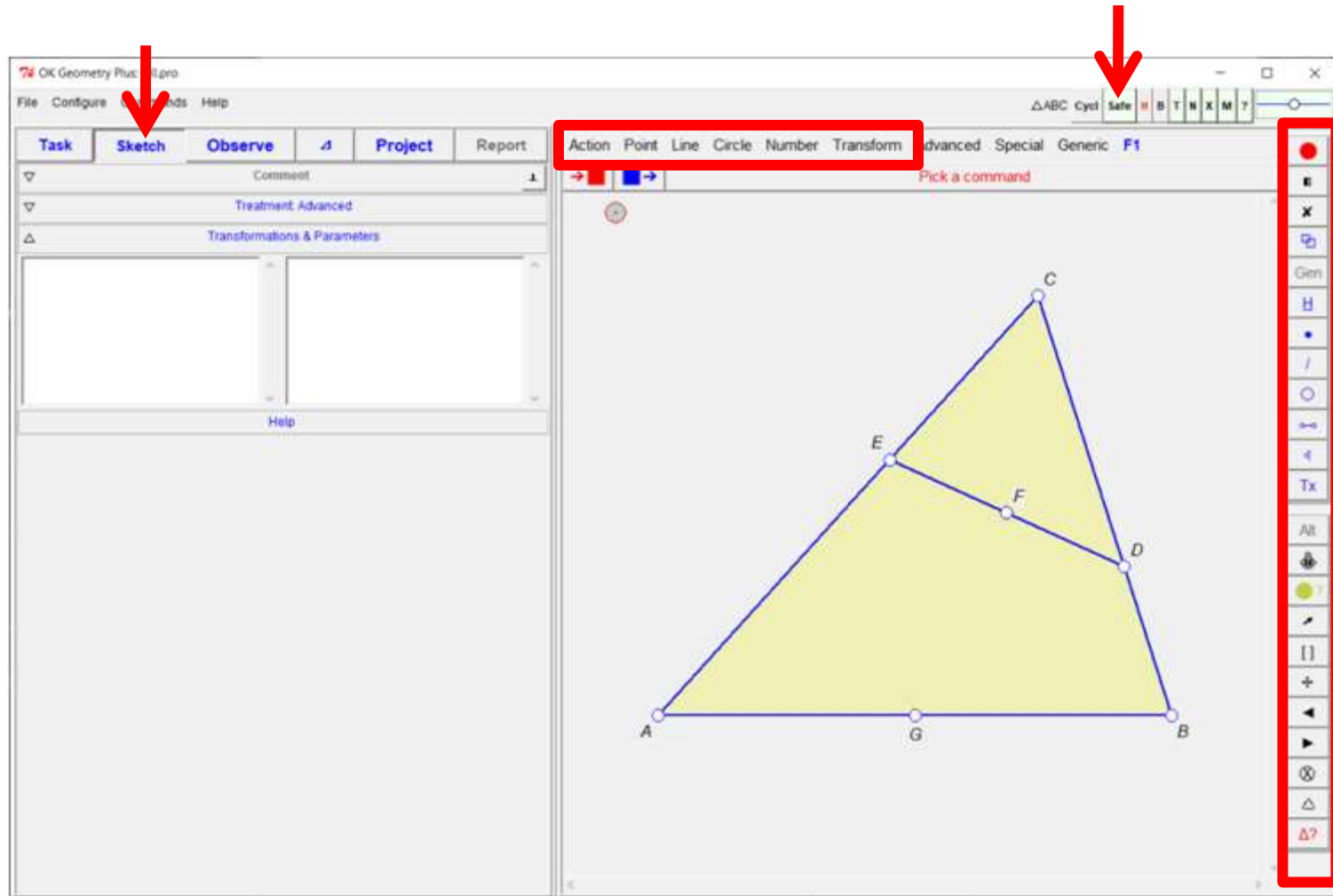
How to observe?













OKG Sketch Editor








- Configuration vs. construction
- OKG observation requires (several) ‘exact’ configurations.
- Sketch Editor creates
 - Constructions
 - Difficult objects
 - Implicit constructions (configurations)
 - Configurations by optimisation

OKG Sketch Editor







OKG Sketch Editor – common buttons

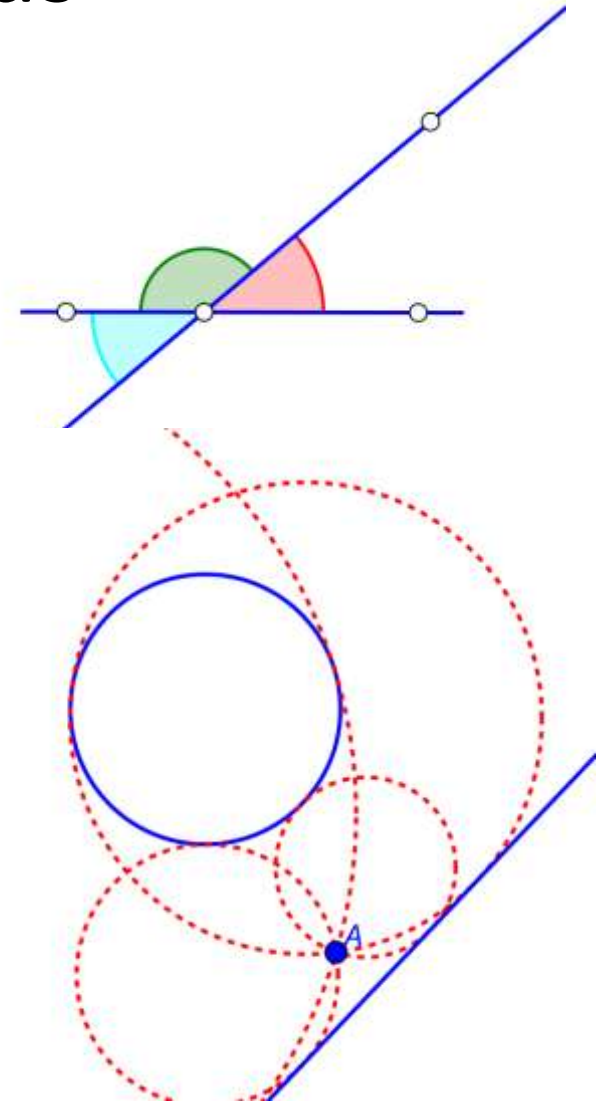
	Shape objects
	Hide objects
	Delete objects
	(Scenes,...)
Gen	(Generic view)
	Label points
	Point, Intersection, Midpoint
	Line, Line 2 obj
	Circle, Circle 3 obj , ...
	Segment, Perp.seg., Polyline
	Angle, Various decorations
Tx	Text

Safe	Safe objects
Alt	Alternative objects
	Anchor
	(Mark Unknown)
	Drag point
[]	Zoom view ...
+/-	Move view
	Undo
	Redo
	Redefine
	(Declare cyclic)
	(Triangle analysis)

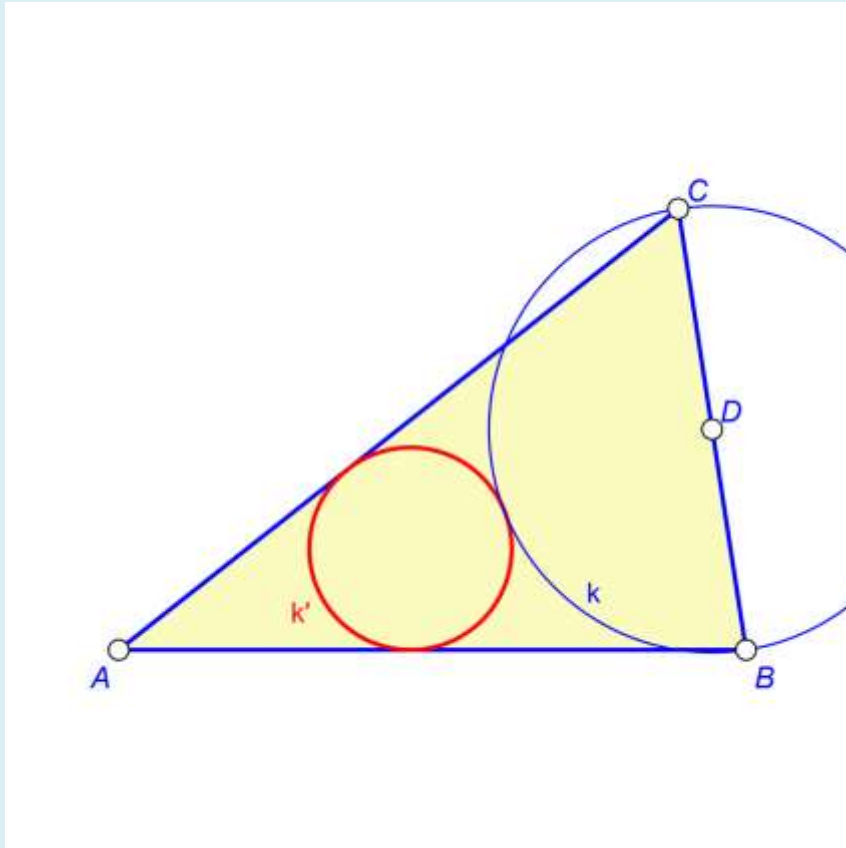
F8 – Help
ON/OFF

OKG Sketch Editor – special commands

	Safe ON	When necessary, segments are treated as lines, arcs as circles.
	Alt (try mouse scroll)	Press repeatedly for alternative solutions.
	Anchor (otrymouse scroll)	Press repeatedly for different ways of representation of objects,
	Line 2 objects + Alt (try mouse scroll)	Line defined by 2 objects in terms of 'passing through', 'is parallel', 'is tangent', 'is radical axis'.
	Circle 3 objects + Alt (try mouse scroll)	Circle defined with 3 objects in terms of 'passing through', 'is tangent'.

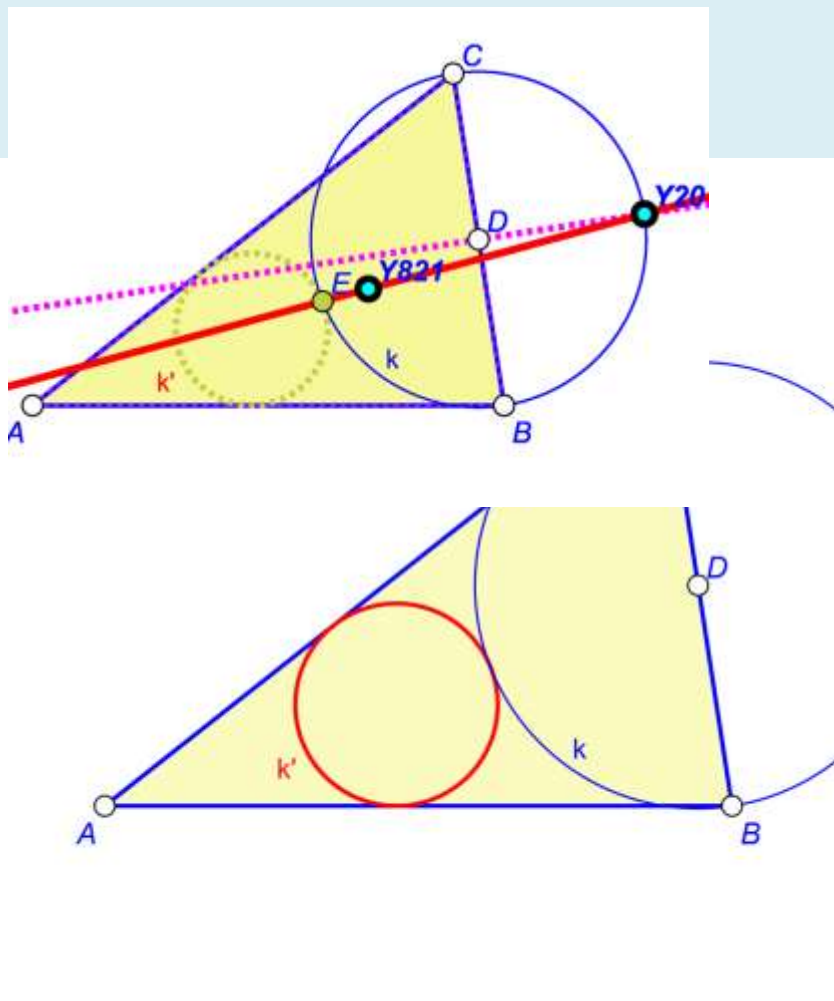


A 'difficult' circle



- ABC – an acute triangle
- $k = k(D,B)$ – circle with diameter BC
- k' – a circle inscribed in the 'triangle' bound by AB , k and CB .
- Analyse the circle k' .

A 'difficult' circle



$$1 \cdot ra \cdot a + ra \cdot b + ra \cdot c + \frac{1}{2} \cdot a^2 - \frac{1}{2} \cdot b^2 + b \cdot c - \frac{1}{2} \cdot c^2 - S = 0$$

$$1 \cdot ra \cdot a + ra \cdot b + ra \cdot c + \frac{1}{2} \cdot a^2 - a \cdot ri - \frac{1}{2} \cdot b^2 + b \cdot c - b \cdot ri - \frac{1}{2} \cdot c^2 - c \cdot ri = 0$$

$$1 \cdot ra \cdot a \cdot (r \cdot \cos(A)) + ra \cdot b \cdot (r \cdot \cos(A)) + ra \cdot c \cdot (r \cdot \cos(A)) + \frac{1}{4} \cdot a^3 + \frac{1}{2} \cdot a^2 \cdot (r \cdot \cos(A)) - \frac{1}{4} \cdot a \cdot b^2 - \frac{1}{4} \cdot a \cdot c^2 - \frac{1}{2} \cdot b^2 \cdot (r \cdot \cos(A)) + b \cdot c \cdot (r \cdot \cos(A)) - \frac{1}{2} \cdot c^2 \cdot (r \cdot \cos(A)) = 0$$

$$(-\frac{1}{2}) \cdot ra^2 \cdot (r \cdot \cos(A)) - \frac{1}{4} \cdot ra^2 \cdot (r \cdot \cos(B)) - \frac{1}{4} \cdot ra^2 \cdot (r \cdot \cos(C)) + \frac{1}{4} \cdot ra^2 \cdot ri + ra \cdot (r \cdot \cos(A)) \cdot ri + \frac{1}{2} \cdot ra \cdot (r \cdot \cos(B)) \cdot ri + \frac{1}{2} \cdot ra \cdot (r \cdot \cos(C)) \cdot ri - \frac{1}{2} \cdot ra \cdot ri^2 - \frac{1}{2} \cdot (r \cdot \cos(A)) \cdot ri^2 = 0$$

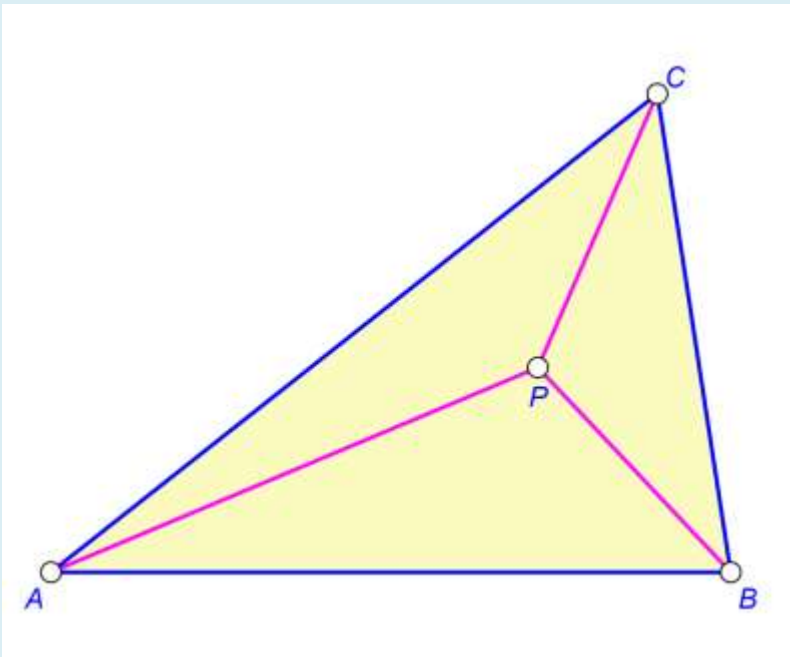
An optimisation problem

ABC – **reference triangle**

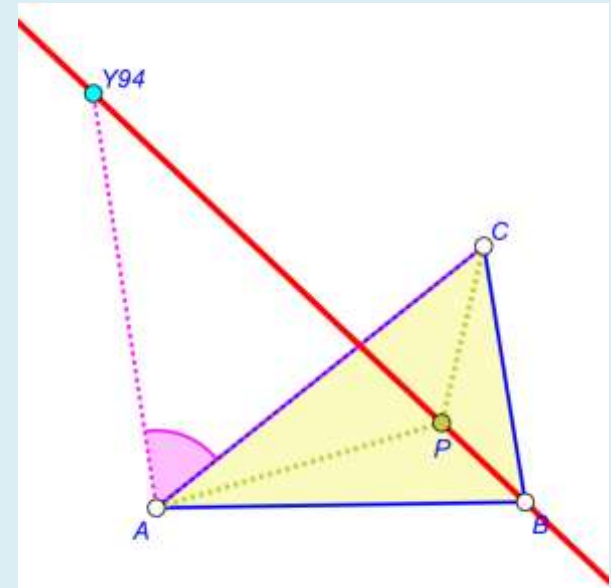
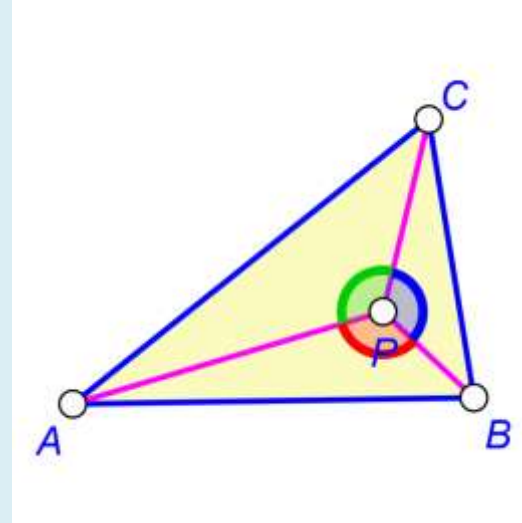
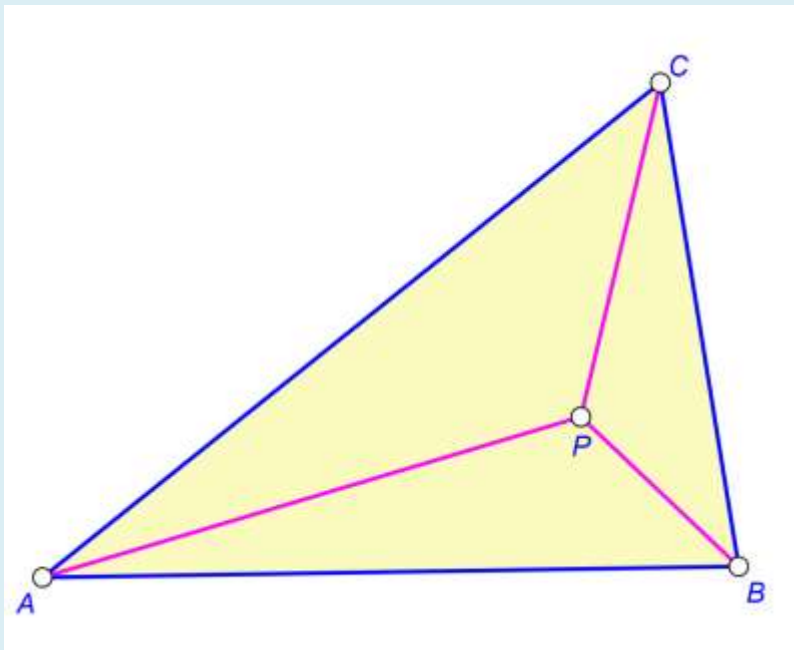
P – point on plane that
minimises

$$|AP| + |BP| + |CP|.$$

Analyse the position of
such a point P.

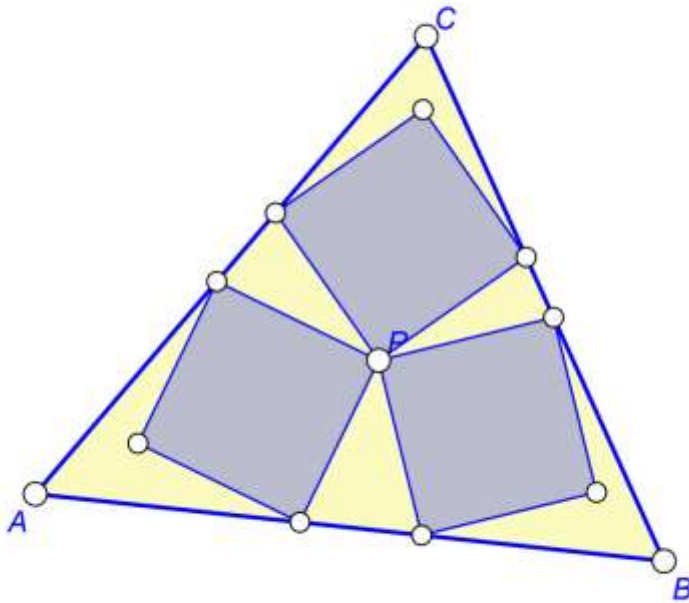


An optimisation problem

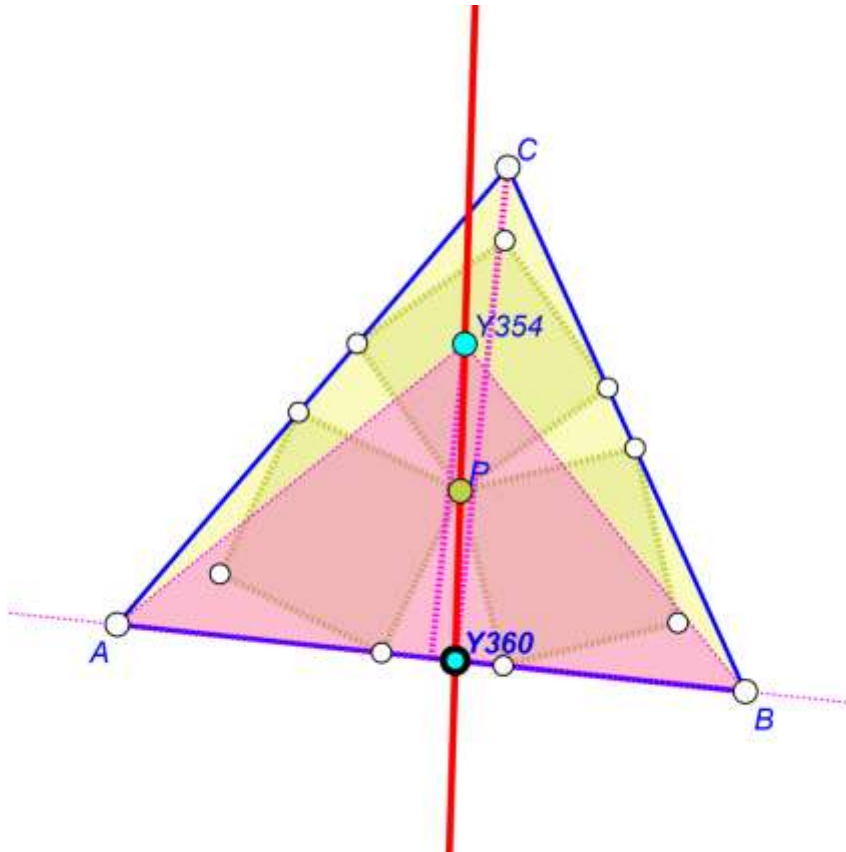


A nice problem

How to inscribe 3 congruent squares into a given triangle ABC as shown in the figure?



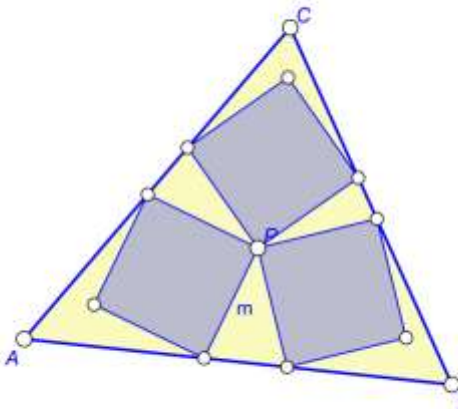
A nice point



- Y354 = Local coordinates $x = 1/2, y = 1/2$; Object(s): A,B
- Y360 = Projection onto line of point; Object(s): AB,C

A nice point

Hypothesise the size m of squares in terms of common triangle quantities.



7% Observe formulae

Mode **Triangle - express a quantity (ratio) in terms of triangle parameters**

Variable, quantity or ratio

m

Transform variables Notation **extended**

Stop at first Strength **4**

Observed formulae can be incorrect or missed !

Ref. triangle = ABC; Observed formula for: m,r,ri,SW,S

$$(-\sqrt{1/2}) * m * r_i * SW + (-\sqrt{1/2}) * m * r_i * S + r * r_i * S = 0$$

Ref. triangle = ABC; Observed formula for: m,a,b,c,ri

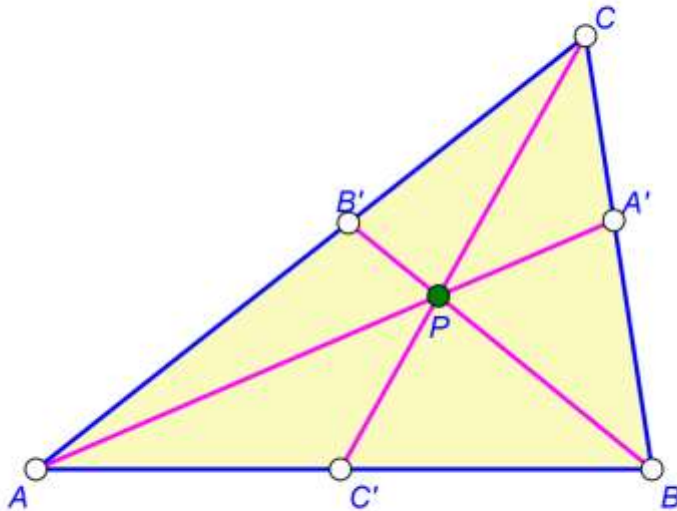
$$(1/2) * m * a^2 + m * a * r_i + 1/2 * m * b^2 + m * b * r_i + 1/2 * m * c^2 + m * c * r_i + (-\sqrt{1/2}) * a * b * c = 0$$

Done

Observe Help Exit

An 'implicit' object

Equal_AA_BB_CC = False

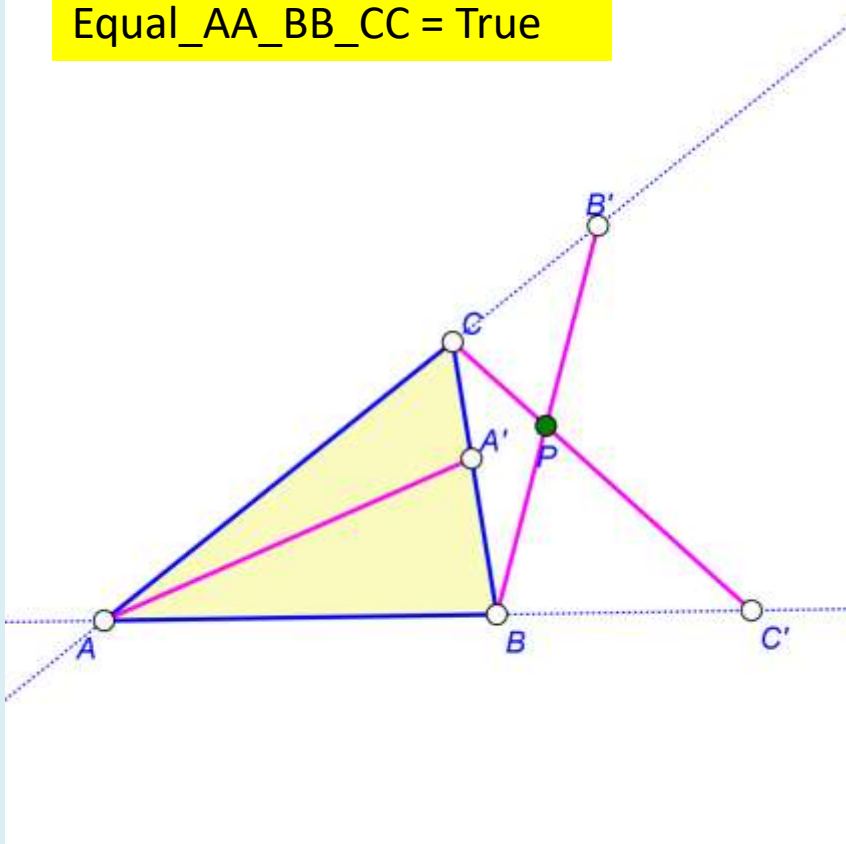


- ABC – a triangle
- P – a point
- AA', BB', CC' – Cevian lines of P in ABC.
- **(AA' ≡ BB' ≡ CC')**

Investigate!

An 'implicit' problem

Equal_AA_BB_CC = True



- ABC – a triangle
- P – a point
- AA', BB', CC' – Cevian lines of P in ABC.
- **AA' ≅ BB' ≅ CC'**

Investigate!

Triangle geometry

- Observe objects wrt. reference triangle
- Drawing triangle objects
- Glossary of triangle objects
- Observing algebraic relations in a triangle

Triangle observation

Triangle centre analysis of P Reference triangle: ABC

Centres X1 - X: 3500 Short centre names **More** **Extensive** **Continue**

The point P contains these ABC related finite points: (1 items, unreliable)
X13: 1st ISOGONIC CENTER (FERMAT POINT, TORRICELLI POINT)

The point P touches these ABC related circles: (1 items, unreliable)
Lester circle

The point P touches these ABC related lines: (1 items, unreliable)
Fermat line

(Possibly transformed) P lays on these ABC related conics: (2 items, unreliab)
P on Evans conic
P on Kiepert hyperbola

Put something in block and right click or use: **Show** **What's** **Glossary**

1st ISOGONIC CENTER (X(13))

Description
 (Right click on the description to activate command and more...)

Special | Triangle centres | Fermat point (1st isogonic point) X(13)

Fermat point (1st isogonic point) X(13)

1 (Ref triangle)

In the triangle ABC let CBA' be the equilateral triangle constructed outwardly on BC. Define B' and C' cyclically. The 1st isogonic point X of the triangle ABC is the point of concurrence of the lines AA', BB', and CC'. This is the Kimberling centre X(13).

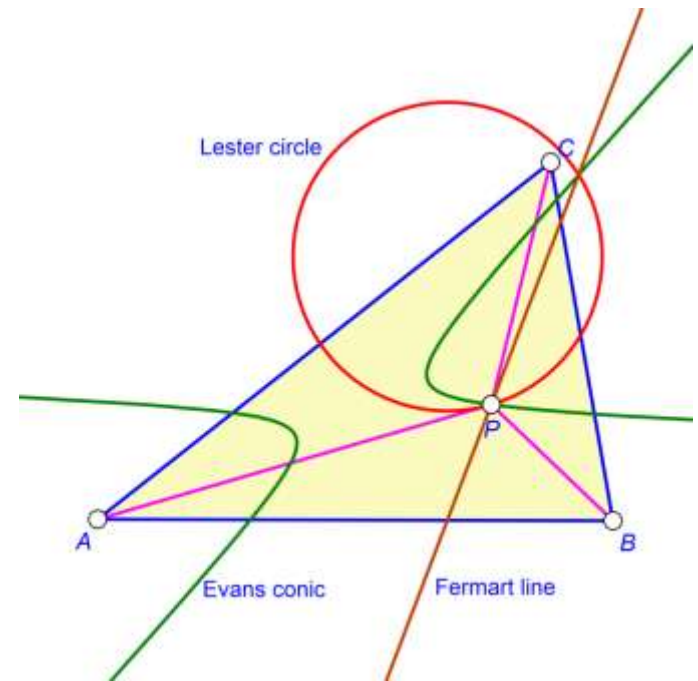
The triangle A'B'C' (not shown) is called the **Fermat outer triangle** of ABC.

If all the angles of $\triangle ABC$ do not exceed 120° then the 1st isogonic point coincides with the **Fermat point**, i.e. the point X that minimises the sum of the distances from X to the vertices of the triangle.

References

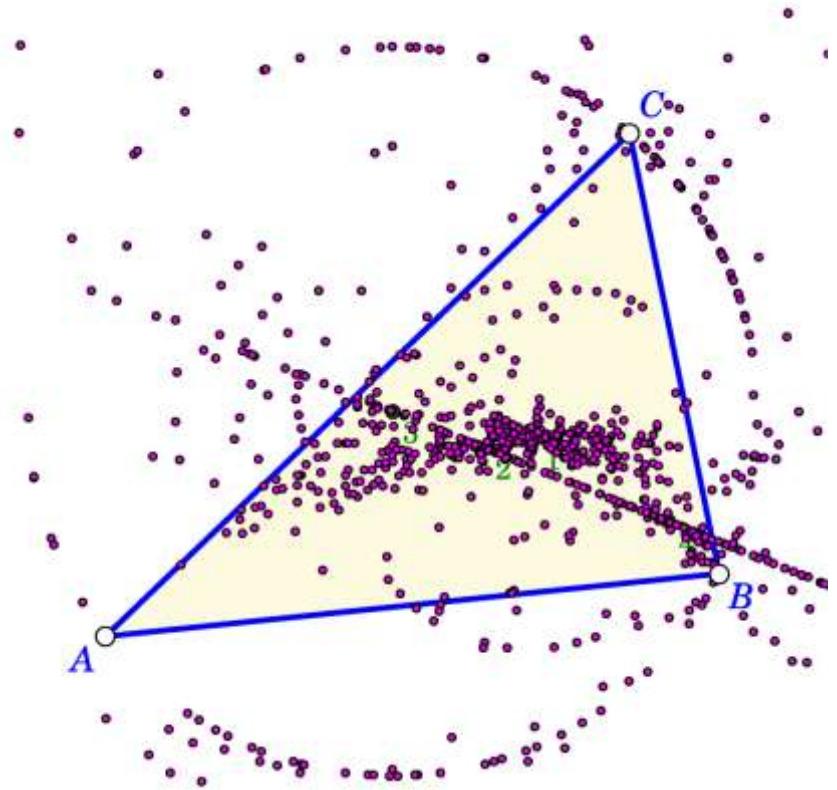
Douillet, L., "Translation of the Kimberling's Glossary into barycentrics", <http://www.douillet.info/~douillet/triangle/glossary/glossary.pdf>

Kimberling, C., "Encyclopedia of Triangle"



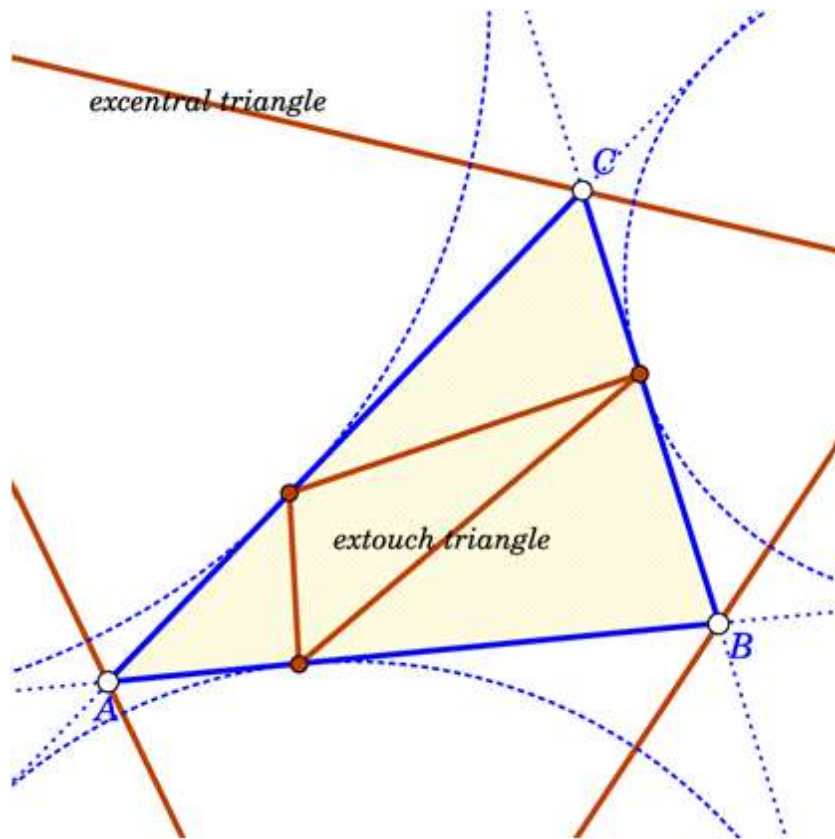
Triangle centres and transformations

- >50.000 centres
- >30 transformations
- ~500.000 transformed centres
- millions of lines connecting the centres

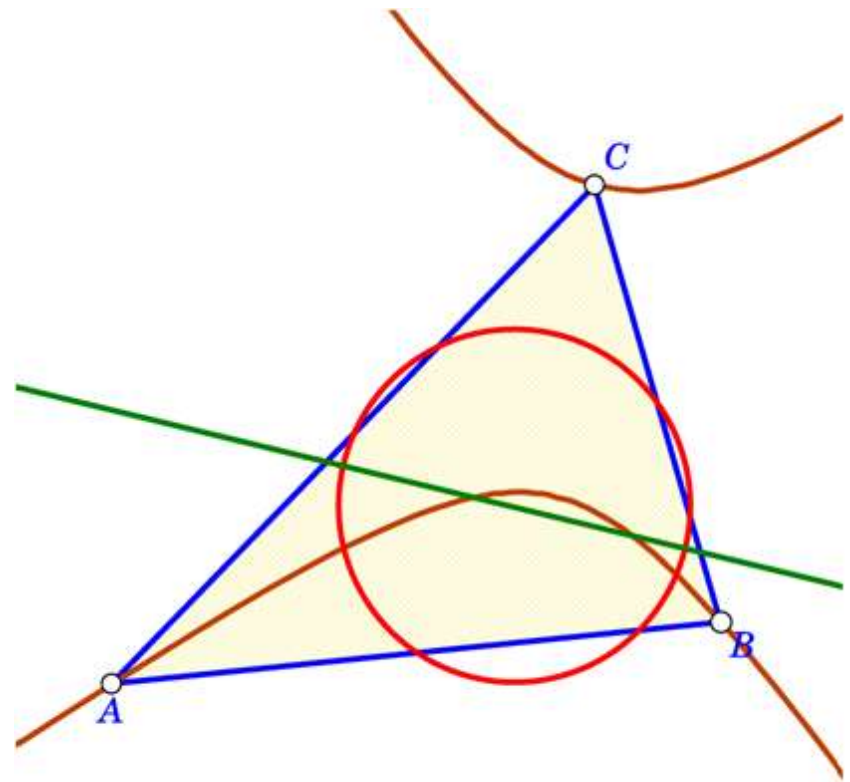


Triangle objects

~ 230 considered triangles →
>2000 lines, >6000 circles

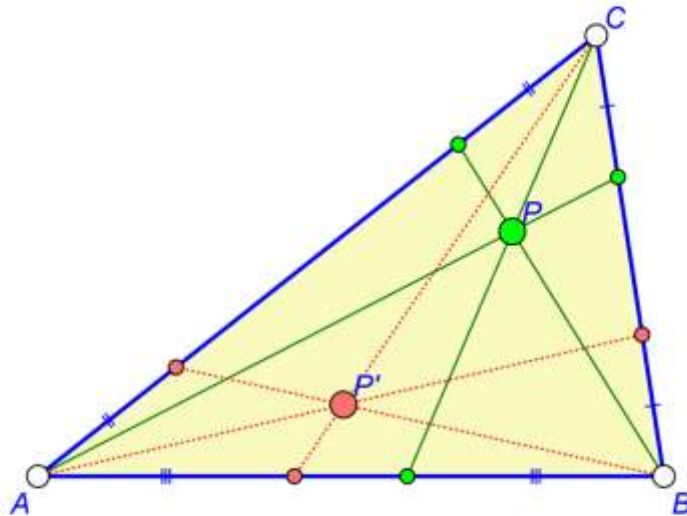


~ 30 considered lines
~100 circles, ~40 conics, ~1300 cubics

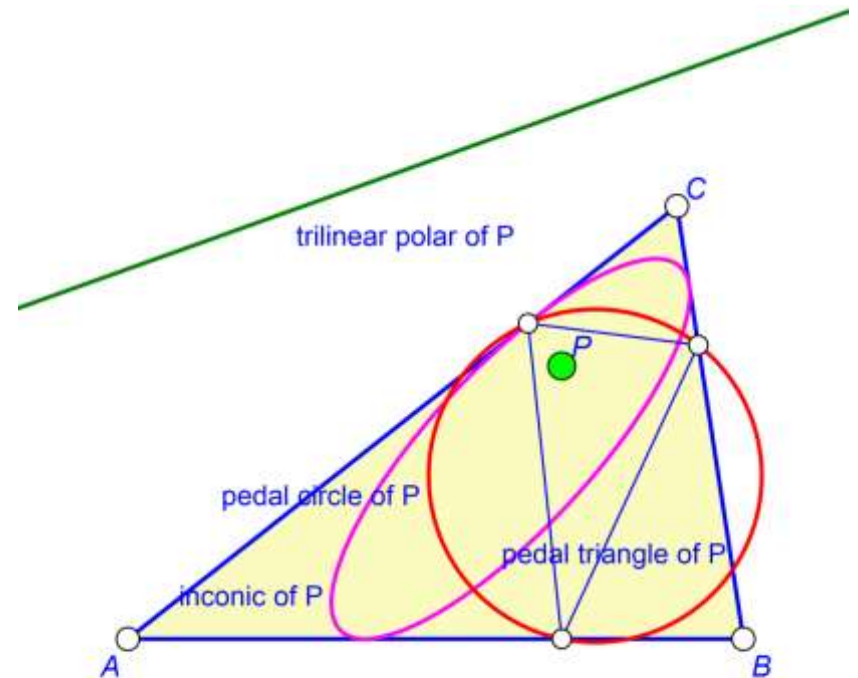


Triangle centres and transformations

Triangle transformations
(e.g. isotomic conjugation)



Triangle-Point objects



Triangle objects

Advanced Special Generic **F1**

Pick a cor

- Triangle centres ...
- Triangle derived objects ...
- Triangle/Point derived objects ...
- Triangle/Point/Point derived objects ...
- Objects by triangle centres ...
- General triangle derived point ...
- Various constructions ...

- Set reference triangle ...
- A triangle
- Declare cyclic objects
- Cyclic construction

- Observe formulae in triangle
- Centre analysis

Triangle derived objects

Output: Line Circle Triangle Conic Cubic abc

Choose from

- K001 NEUBERG CUBIC
- K002 THOMSON CUBIC
- K003 McCAY CUBIC
- K004 DARBOUX CUBIC**
- K005 NAPOLEON CUBIC
- K006 ORTHOCUBIC, pK(X6, X4)
- K007 LUCAS CUBIC, pK(X2, X69)
- K008 DROUSSENT CUBIC

Description

(Right click on the description to activate command and more...)

Special | Triangle derived objects | K004 DARBOUX CUBIC

K004 DARBOUX CUBIC

1. (Ref. triangle)

Given is a triangle **ABC**. For a point **P** let **A'B'C'** be the pedal triangle of **P** in **ABC**.

The **Darboux cubic** of **ABC** is the locus of points **P** for which the pedal triangle **A'B'C'** of **P** is perspective to the triangle **ABC**.

Darboux cubic is a self-isogonal cubic with pivot **X(20)**.

References

Filter

Go Comments

See picture

OK Cancel Import construction

Glossary

Glossary ✕

Enter approximate entry

pedal

OK Cancel

Commands **Special (triangle)** ETC CTC

Item on 'pedal' or similar

- Pedal (triangle)
- Antipedal (triangle)
- Pedal circle
- Apedal conic
- Simpedal point**
- Antipedal line* (line)
- Pedal antipodal perspecter
- pedal/Cevian similarity point (X1138)

Description

(Right click on the description to activate command and more...)

Special | Various constructions | Point | **Simpedal point**

Simpedal point

- 1st triangle
- 2nd triangle

Given are triangles $\triangle ABC$ and $\triangle A'B'C'$. There exists a point P so that the pedal triangle $\triangle A''B''C''$ of P in $\triangle ABC$ is (directly) similar to $\triangle A'B'C'$.

The point P is called the **simpedal point** of $\triangle ABC$.

Note: $\triangle A'B'C'$ should not be inversely similar

Explain term
Triangle glossary
Copy description to clipboard
Execute command (in Sketch editor)

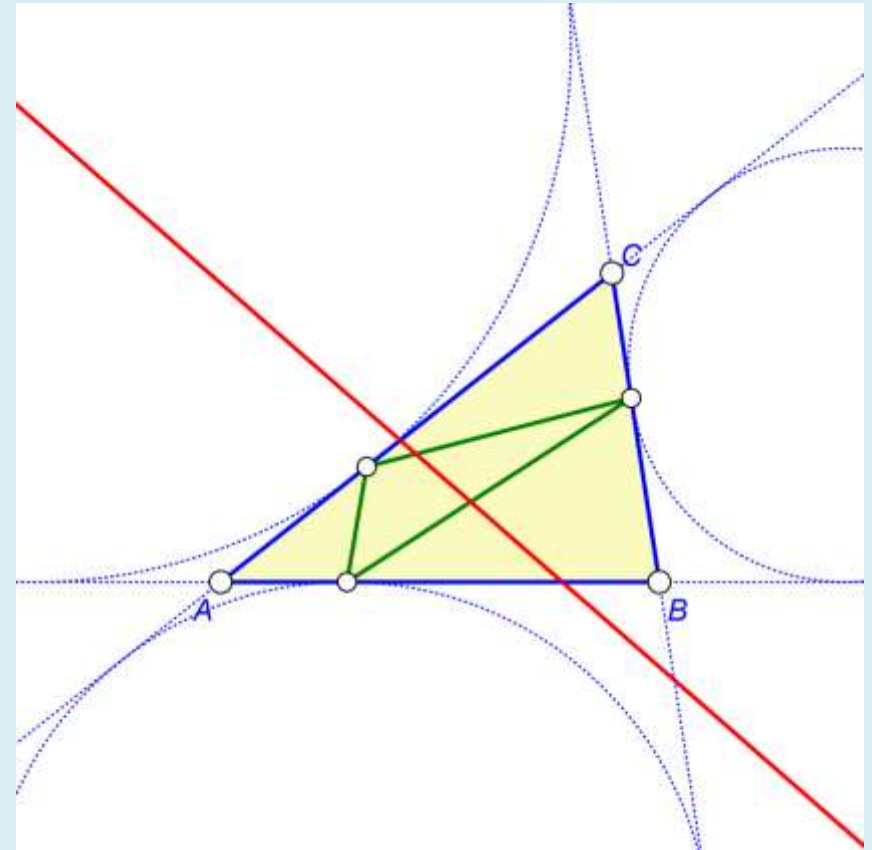
New search Go

See picture

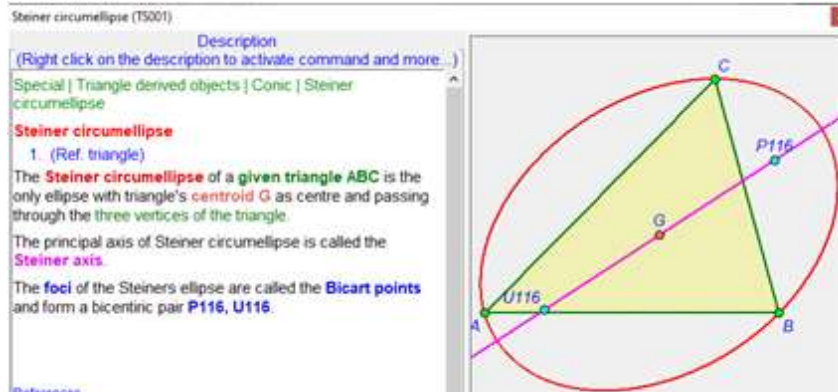
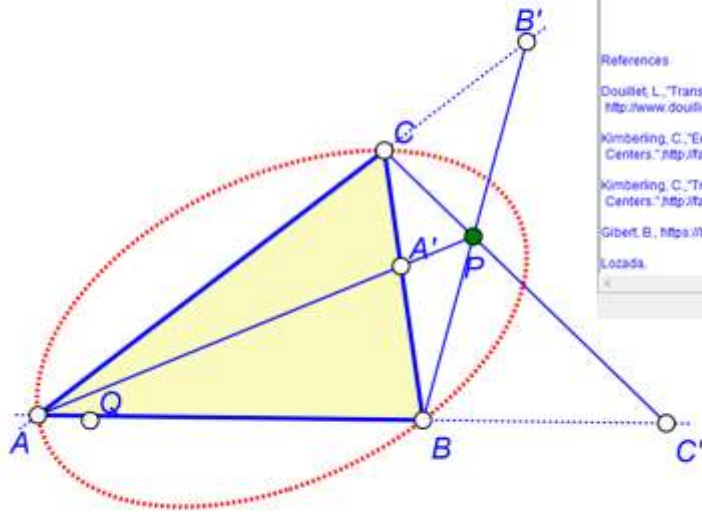
OK Cancel Import construction

Triangle objects

- Given is a triangle ABC.
- Draw the Euler line of the extouch triangle of ABC.



Congruent Cevians



Triangle centre analysis of P Reference triangle: ABC

Centres X1 - X 16342+bic Short centre names **More** **Extensive** **Continue**

The point P contains these ABC related finite points: (1 items, unreliable)
P116: 1st REAL FOCUS OF STEINER CIRCUMELLIPSE

Transformed P matches these ABC related points: (1 items, unreliable)
P = P116: 1st REAL FOCUS OF STEINER CIRCUMELLIPSE

The point P touches these lines of the triangles related to ABC: (4 items, unreliable)
 Steiner axis of Medial triangle
 Steiner axis of Antimedial triangle
 Steiner minor axis of Neuberg Triangle
 Steiner axis of 1st Brocard triangle

Mirroring analysis of P for ABC related lines: (1 items, unreliable)
P = Mirror wrp to Steiner minor axis of U116: 2nd REAL FOCUS OF

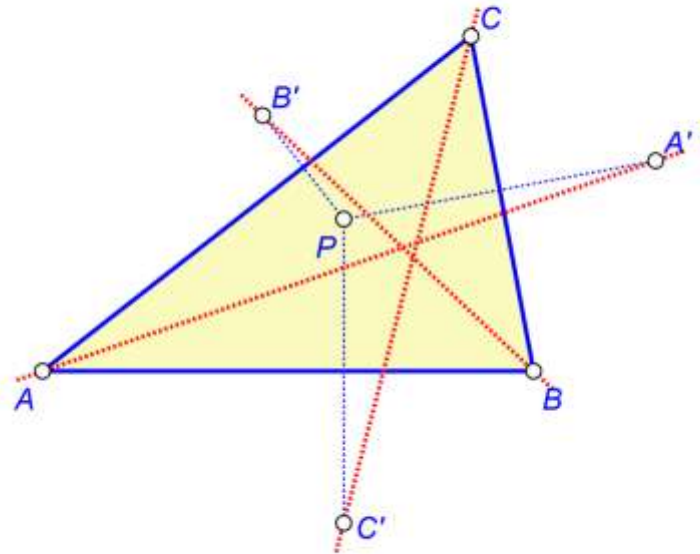
The point P touches these ABC related lines: (1 items, unreliable)
Steiner axis

The point P lays on lines through these finite centres of ABC: (1 items, great ca
 Line:
X2:CENTROID
X1341:EXSIMILICENTER/CIRCUMINFLUENCE/BROCARD CIRC

Put something in block and right click or use: **Show** **Whats** **Glossary**

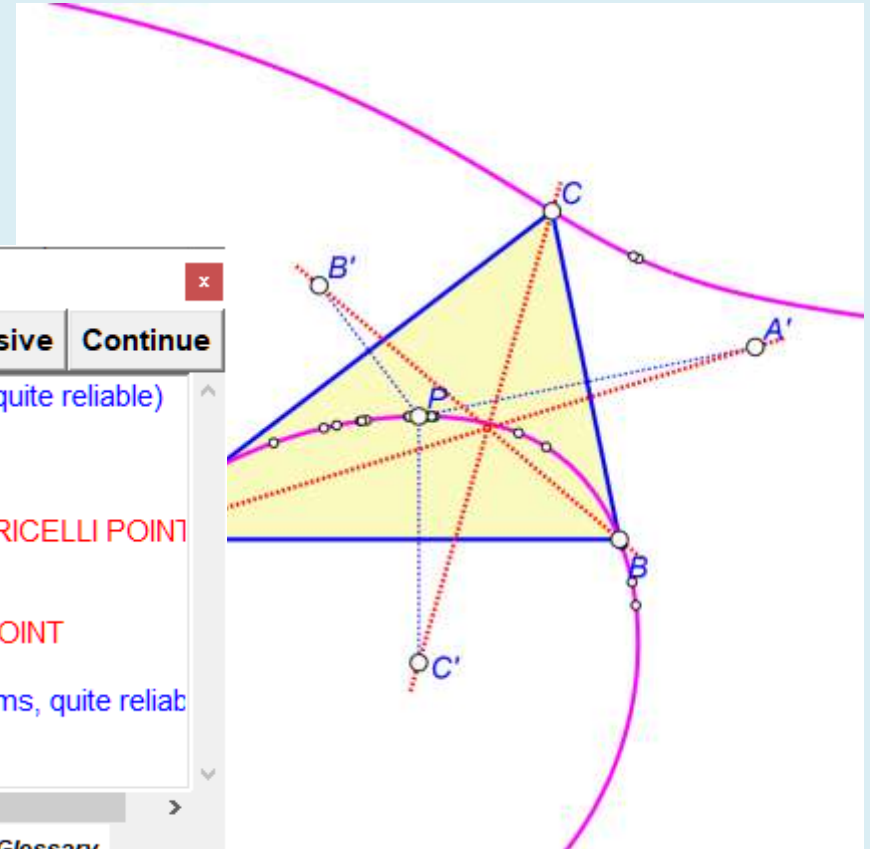
Example of a triangle locus

- A', B', C' are the mirror images of a point P in the sides of triangle ABC .
- For what points P are the lines AA', BB', CC' concurrent?



Example of a triangle locus

- A', B', C' are the mirror images of a point P in the sides of triangle



Triangle centre analysis of * Reference triangle: ABC

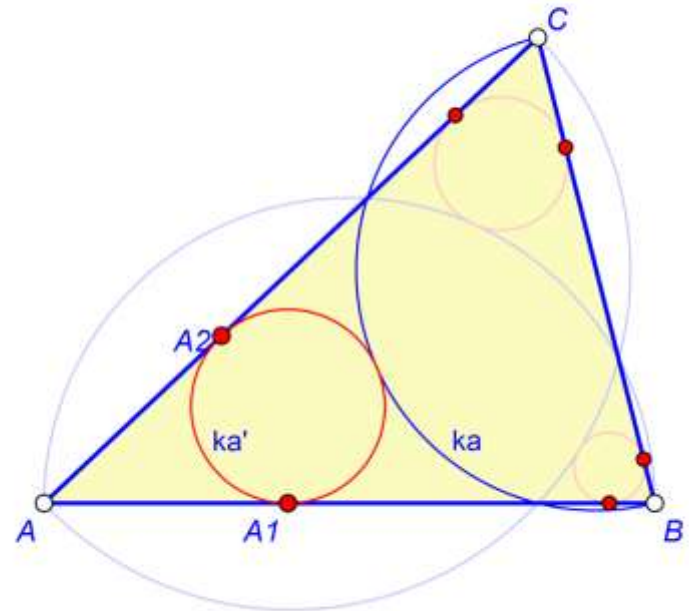
Centres X1 - X Short centre names **More** **Extensive** **Continue**

- The cubic contains these ABC related finite points: (23 items, quite reliable)
 - X1:INCENTER
 - X3:CIRCUMCENTER
 - X4:ORTHOCENTER
 - X13:1st ISOGONIC CENTER (FERMAT POINT, TORRICELLI POINT)
 - X14:2nd ISOGONIC CENTER
 - X15:1st ISODYNAMIC POINT
 - X74:ISOGONAL CONJUGATE OF EULER INFINITY POINT
 - ...
- The cubic * appears to coincide with ABC triangle cubic: (1 items, quite reliable)
 - (*) = K001 NEUBERG CUBIC (?)

Put something in block and right click or use: **Show** **What's** **Glossary**

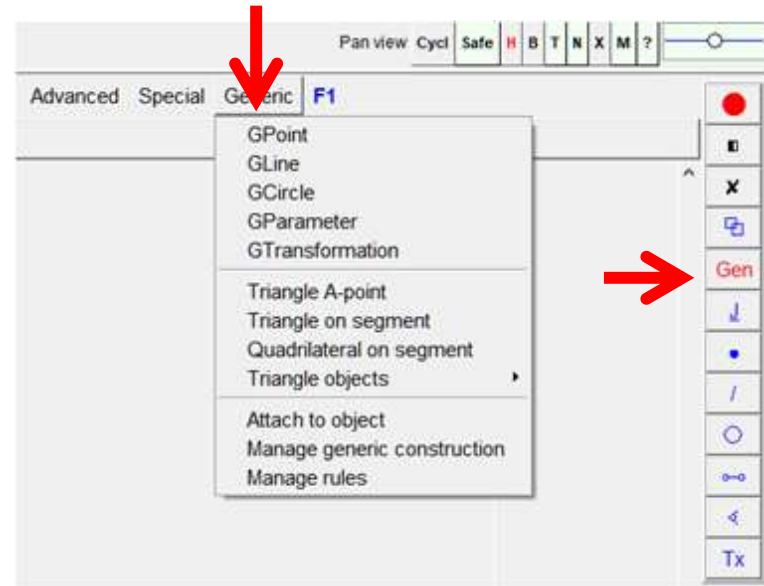
Cyclic constructions

- ABC – an acute triangle
- ka – the inwards semicircle on BC
- ka' – the smallest of circles touching AB , AC , and (externally) ka
- kb' , kc' – defined cyclically
- Investigate the points of contact of ka' , kb' , kc' with the sidelines of ABC .



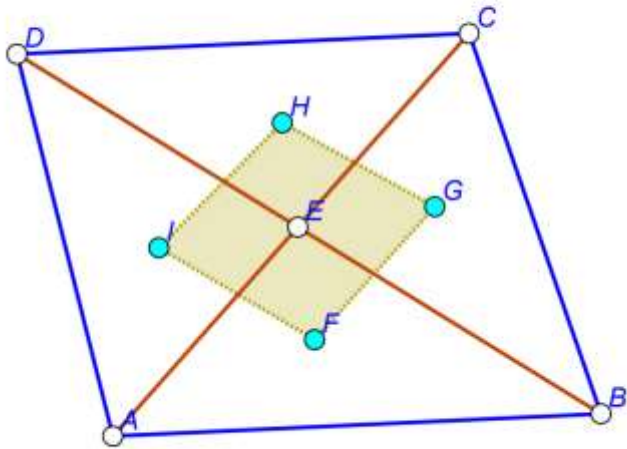
Generic constructions

- Generic constructions are constructionally isomorphic families of dynamic constructions.
- Generic constructions appear and behave like ordinary constructions, in which some construction steps consist of rules (i.e. groups of isomorphic operations).



All resulting constructions can be visualised, analysed, checked for properties, etc. at the same time.

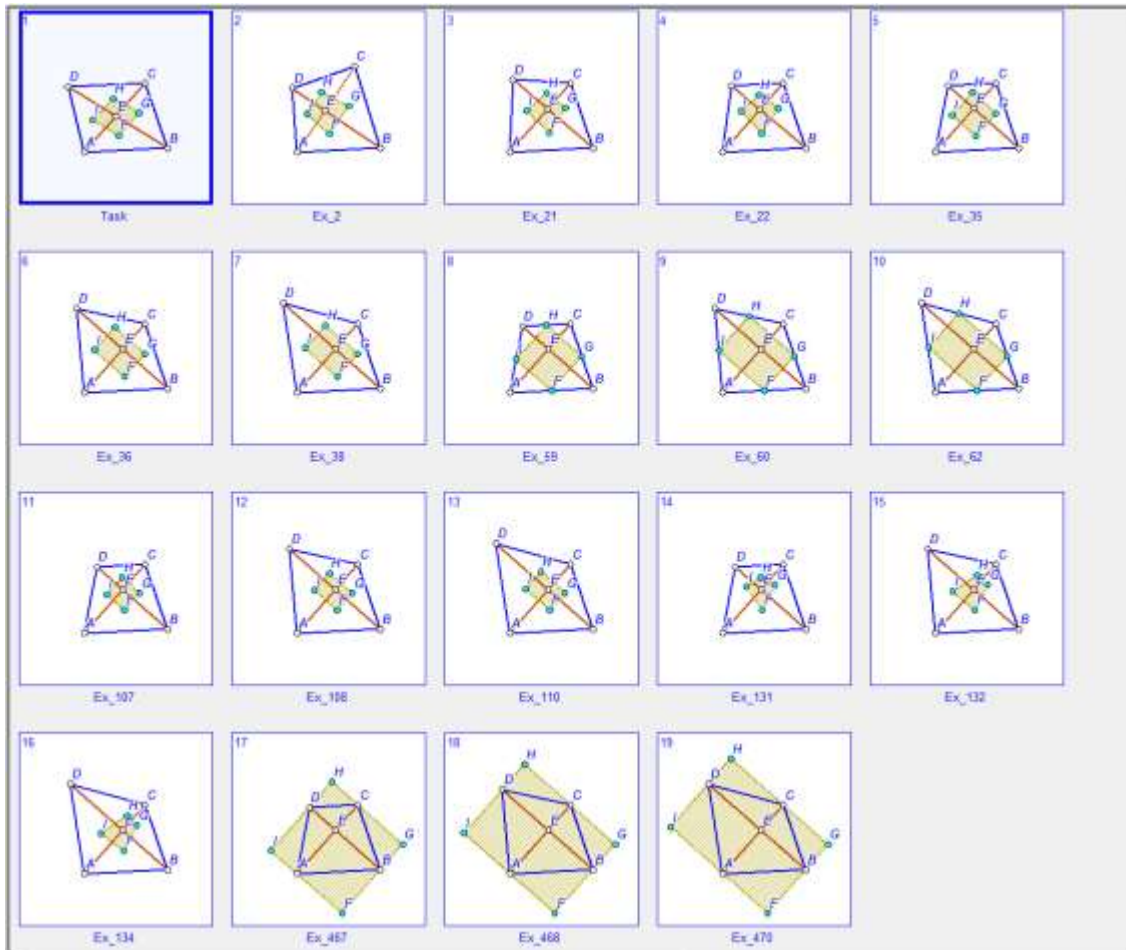
Generic constructions – constructionally isomorphic configurations



1. ABCD – a **trapezium**
2. $E = AC \cap BD$
3. F, G, H, I **incentres** of the 4 triangles (ABE, BCE, CDE, DAE)

1. Quadrilateral →
Random,
bicentric, cyclic,
equidiagonal,)

3. incentres →
incentre, centroid,
circumcentre,
orthocentre, ...



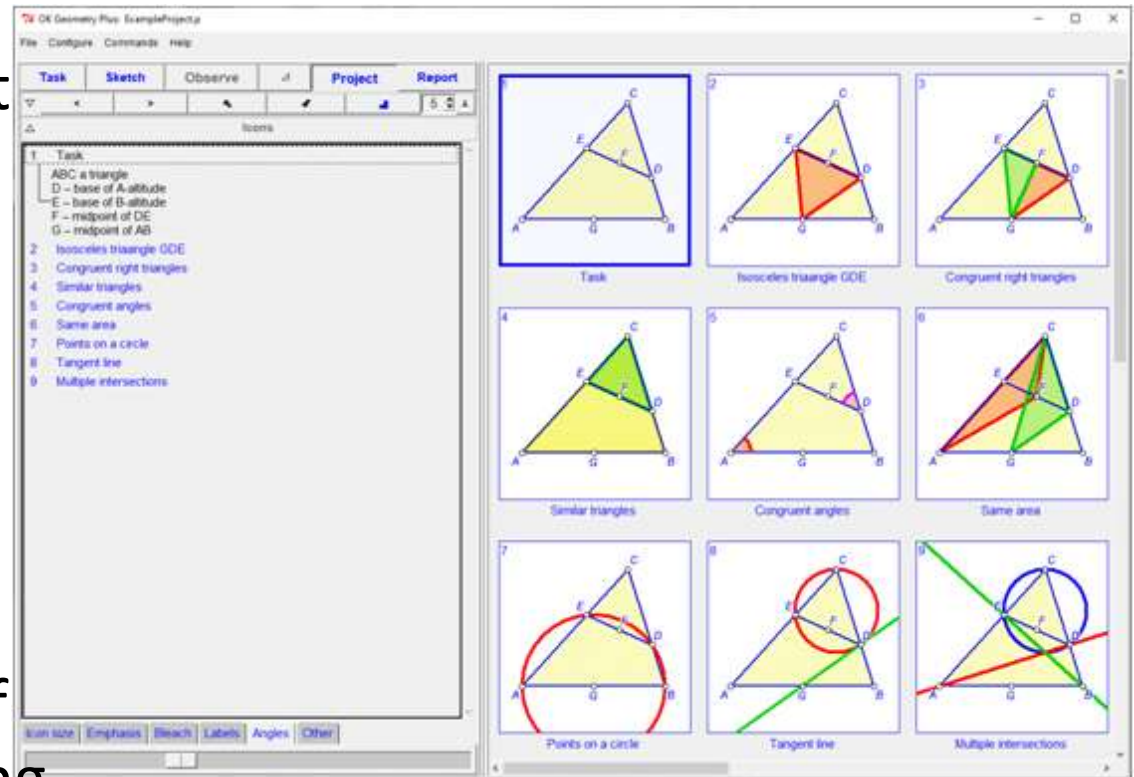
Shaded 4-laterals are cyclic in 43 cases out of 485 checked,

e.g.:

- incentres for bicentric quadrilaterals,
- 9-point centres for Pythagorean quadrilateral...

Projects

- You can collect constructions, parts of constructions, results, etc. into a project.
- A project may contain related constructions, observed properties, a proof of a claim, a proving task, etc.



Saving properties

The screenshot shows the Geometry Plus interface with a geometric construction of a triangle ABC and its medians AD , BE , and CF intersecting at point G . The construction is highlighted in yellow. The left sidebar shows the 'Observed properties' list, which includes:


- congruent segments (2)
- congruent segments (relation) (5)
- ratio of distances (13)
- sum of lengths (relation) (50)
- congruent angles (13)

The 'congruent angles' list includes:

- $\angle BAC = \angle CEA = \angle ADF = \angle BCD$
- $\angle ABC = \angle CEA = \angle ADF = \angle BCD$
- $\angle ACB = \angle CEA = \angle ADF = \angle BCD$
- $\angle BAC = \angle CEA = \angle ADF = \angle BCD$
- $\angle ABC = \angle CEA = \angle ADF = \angle BCD$
- $\angle ACB = \angle CEA = \angle ADF = \angle BCD$
- $\angle BAC = \angle CEA = \angle ADF = \angle BCD$
- $\angle ABC = \angle CEA = \angle ADF = \angle BCD$
- $\angle ACB = \angle CEA = \angle ADF = \angle BCD$
- $\angle BAC = \angle CEA = \angle ADF = \angle BCD$
- $\angle ABC = \angle CEA = \angle ADF = \angle BCD$
- $\angle ACB = \angle CEA = \angle ADF = \angle BCD$

At the bottom left, a warning dialog box is displayed:

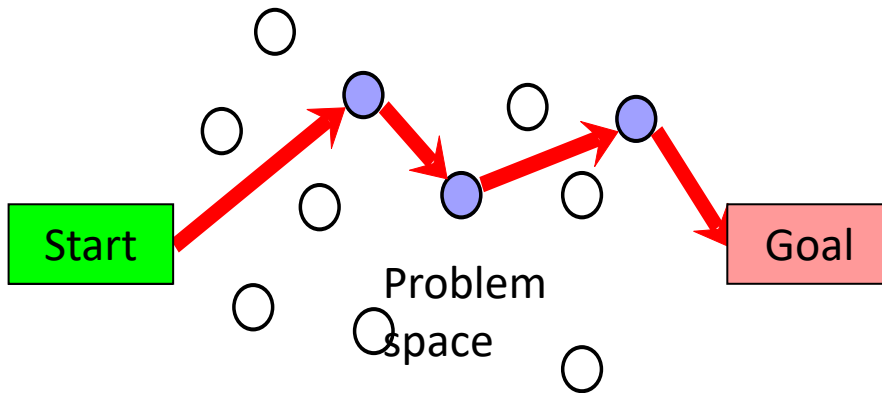
Warning

Use anchor  to select angles, position labels or texts.
Use other commands to delete or modify the added objects.

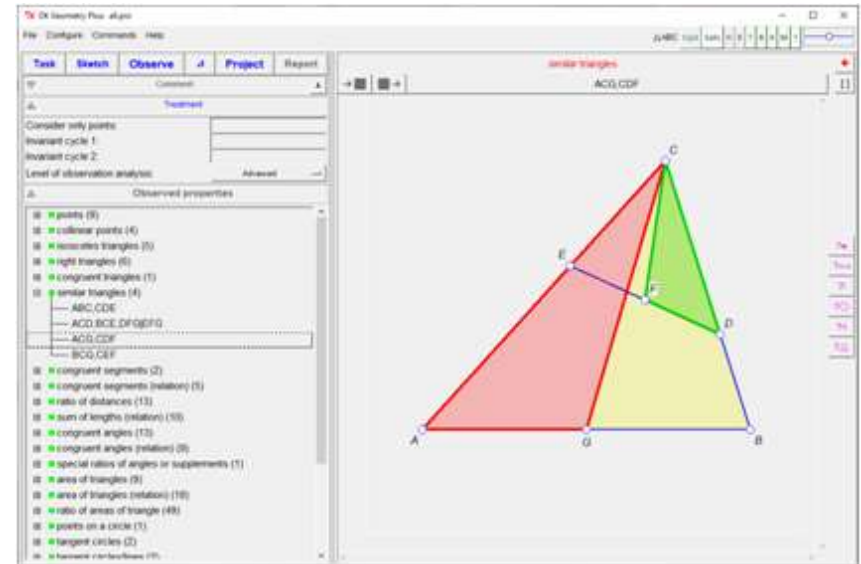
OK

At the bottom right, a 'Icon editor' dialog box is shown, titled 'Congruent angles'. It contains a preview of the triangle construction and a 'Comment' field.

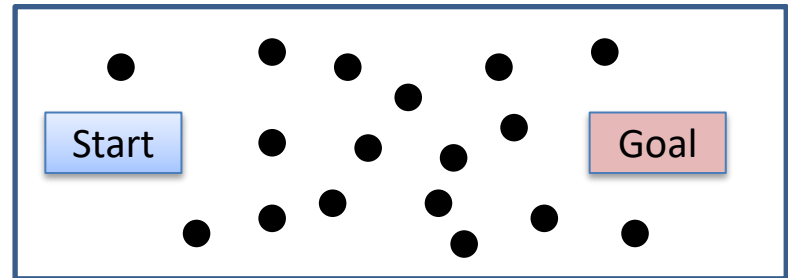
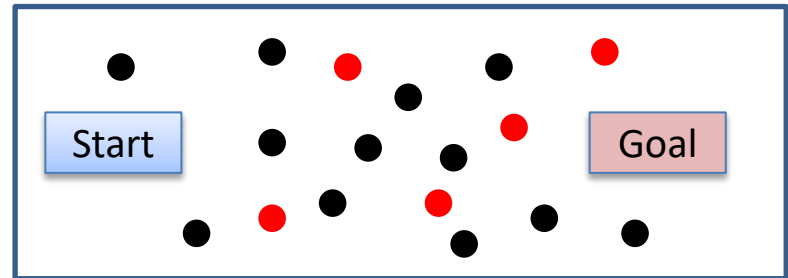
Proving tasks



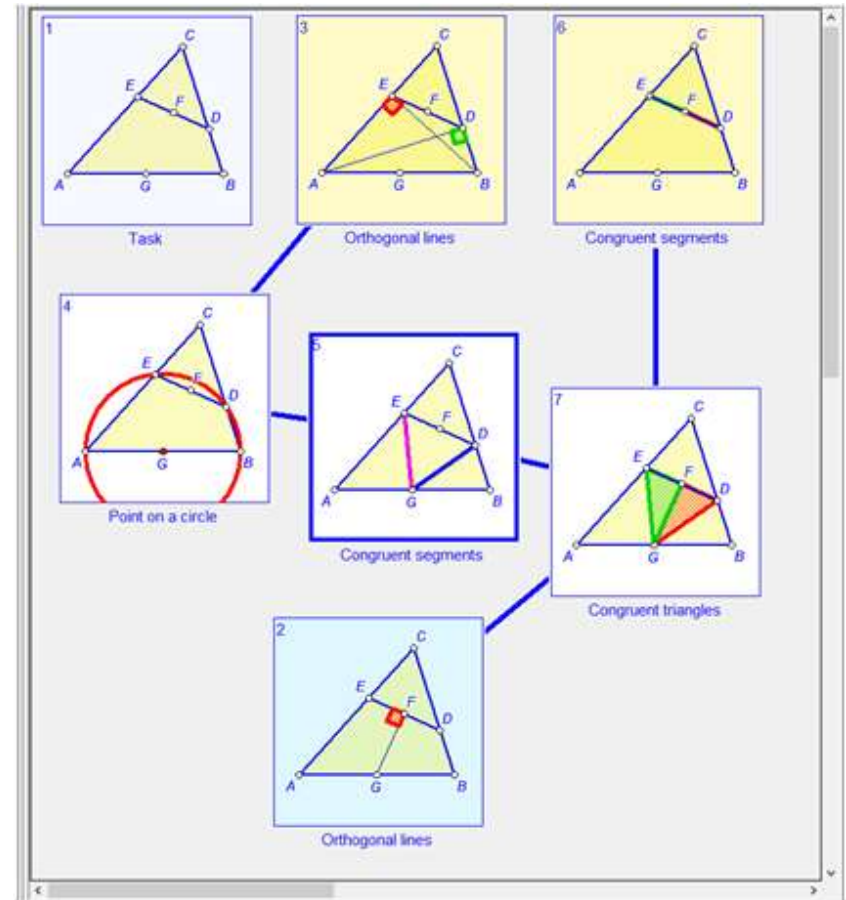
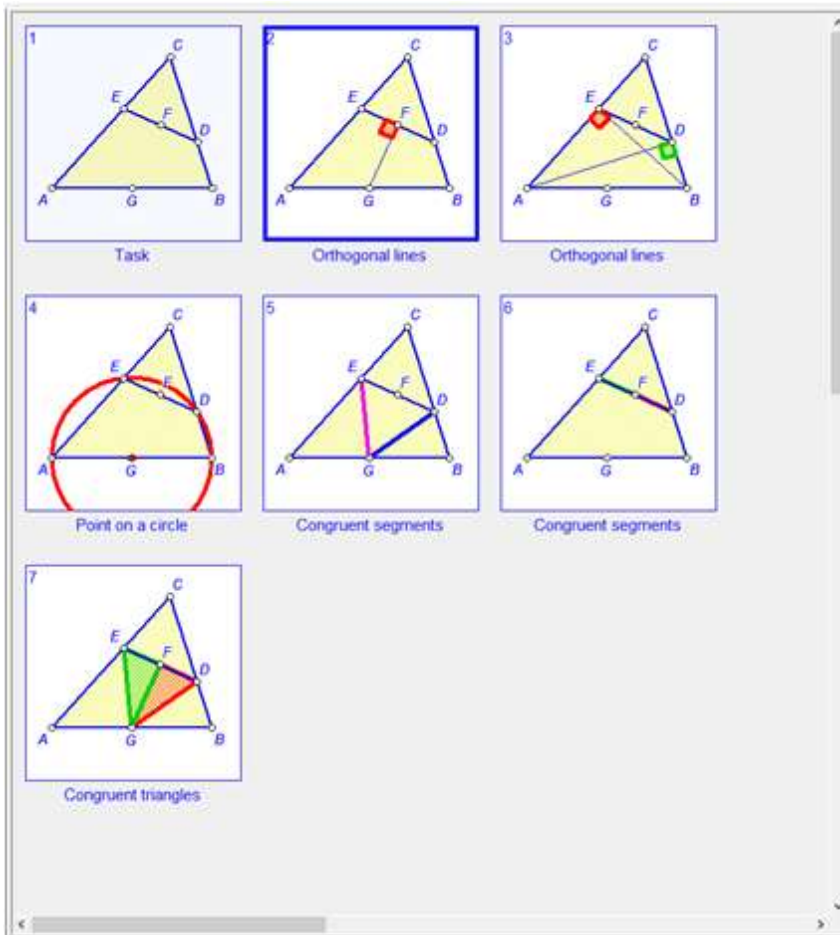
- Observe properties
- Select relevant properties
- Organise the properties
- Provide deductive argumentation

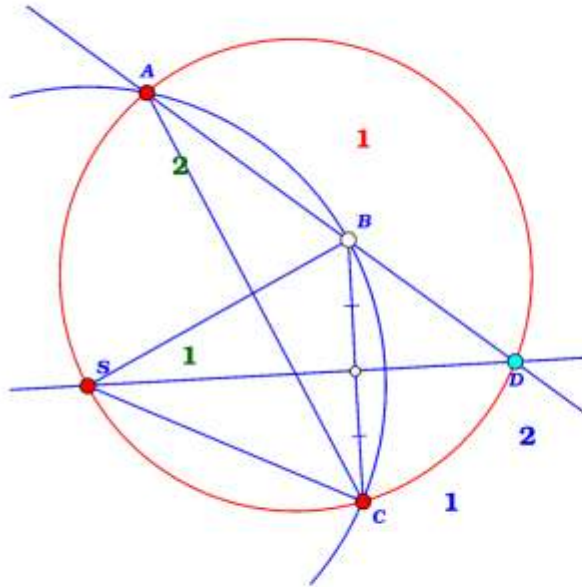


Does a given
problem space
help?



Proving





Task

1 Task

Given is a circle with centre S and three points, A, B, C on its circumference. Let D be the intersection of the line AB and the bisector of the chord BC.

Prove that S, C, D, and A are cocyclic.

Comment:

2 Proof

Definition Let E be the midpoint of BC.

Claim 1 $\angle CSB = 2 \cdot \angle CSD$

Argument 1 First, note that S lays on the bisector of segment BC (since $|SB|=|SC|$). Let E be the midpoint of BC. The triangles AEB and SEC are congruent by sss. Thus

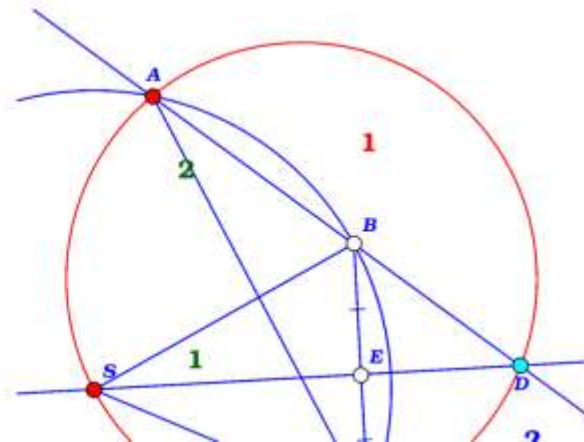
$$\angle CSE = \angle ESB$$







and consequently

$$\angle CSB = 2 \cdot \angle CSD.$$

Claim 2 $\angle CAB = \angle CSD$

Argument 2 The arc BC of the circle $k(S,A)$ spans an inscribed angle $\angle CAB$ and the central angle $\angle CSB$. By a known theorem



	2 <i>Orthogonal lines</i>	
	3 <i>Congruent segments</i>	
	4 <i>Point on a circle</i>	
	5 <i>Congruent segments</i>	
	6 <i>Congruent triangles</i>	
	7 <i>Orthogonal lines</i>	







Author: (Fri Aug 25 19:35:00 2023)
Source: C:\Users\zlatan\zim\Proj\GeoOkDelo\Beograd\nove\ExampleProof.pro

ExampleProject.p

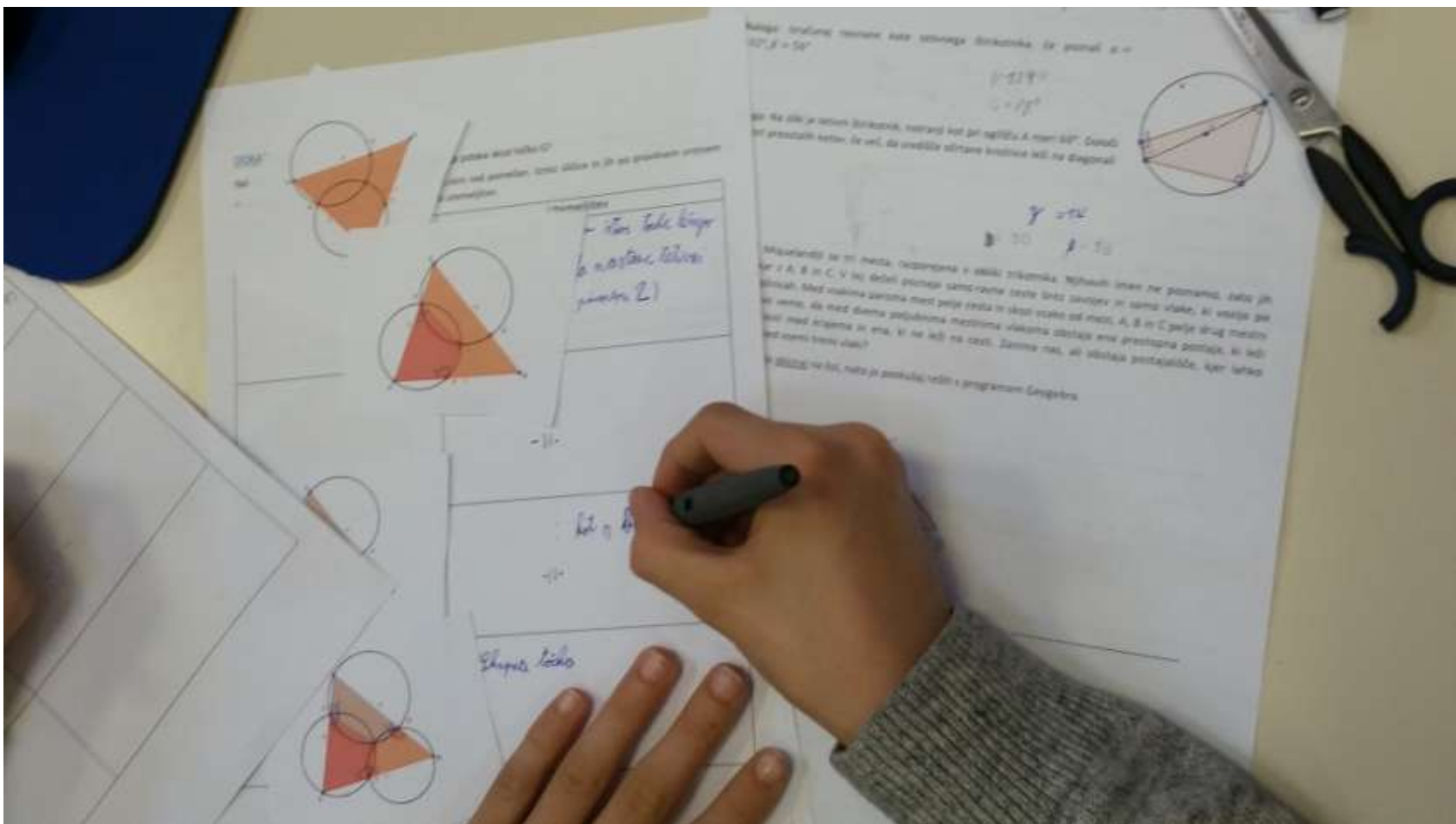
1 Task

Given:
 ABC a triangle
 D – base of A-altitude
 E – base of B-altitude
 F – midpoint of DE
 G – midpoint of AB

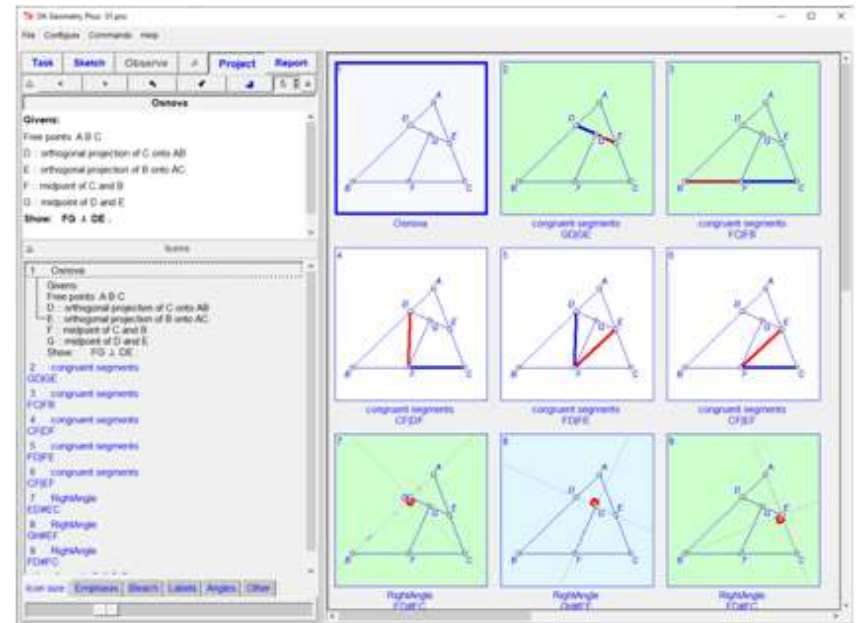
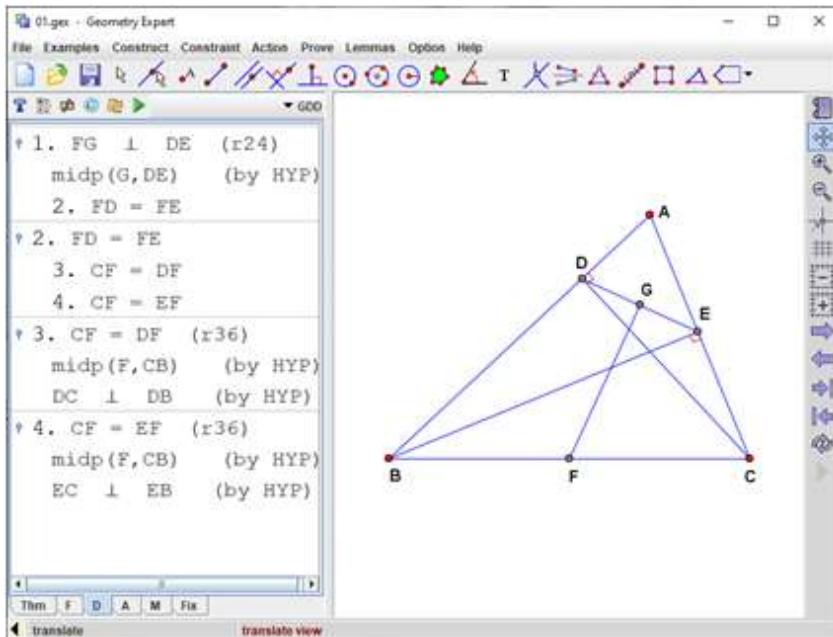
Prove:
 $DE \perp FG$

		
2 <i>Orthogonal lines</i>	3 <i>Congruent segments</i>	4 <i>Point on a circle</i>
		
5 <i>Congruent segments</i>	6 <i>Congruent triangles</i>	7 <i>Orthogonal lines</i>

Notes

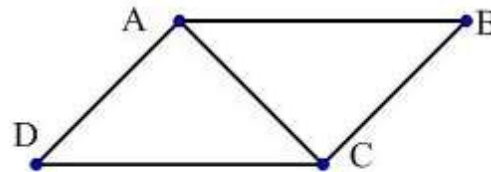


Importing proofs JGEX → OK Geometry

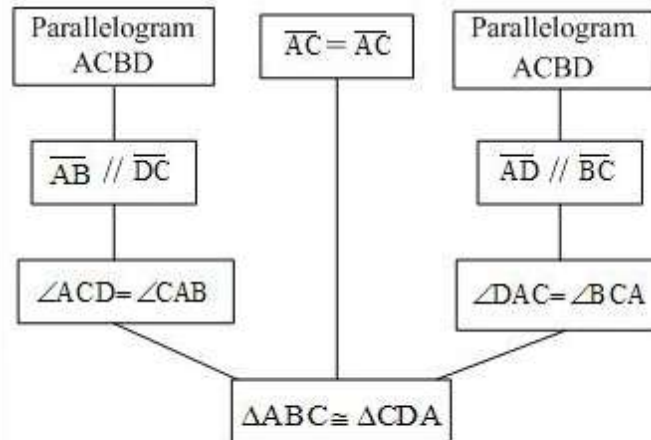


Multiple representations – Mr Geo (Wong, Yin, Yang, Cheng, 2011)

Given: Parallelogram ABCD
with diagonal \overline{AC}
Prove: $\triangle ABC \cong \triangle CDA$



1. ABCD is a parallelogram (Given)
2. \therefore ABCD is a parallelogram, $\therefore \overline{AB} \parallel \overline{DC}$
(Def. of parallelogram)
3. \therefore ABCD is a parallelogram, $\therefore \overline{AD} \parallel \overline{BC}$
(Def. of parallelogram)
4. $\therefore \overline{AB} \parallel \overline{DC}$, $\therefore \angle ACD = \angle CAB$ (Alt. int. angles)
5. $\therefore \overline{AD} \parallel \overline{BC}$, $\therefore \angle DAC = \angle BCA$ (Alt. int. angles)
6. $\overline{AC} = \overline{AC}$ (Reflexive law)
7. $\triangle ABC \cong \triangle CDA$ (ASA)



Justification of claims

Property
 $\angle CSD = \angle CAD$. Why?
 Comment

Emphasize Labels Angles

OK Cancel

1
 $\angle CSD = \angle CAD$. Why?

2
 Point D lays on the circle through A,S,C. Why?

3
 $\angle CSD = \angle DSB$. Why?

4
 $\angle CSD = \angle CAD$. Why?

5
 $\angle CSB = 2 \angle CAB$. Why?

Icon size Emphasize Bleach Highlight Labels Angles

ADG 2023, Belgrade Zlatan Magajna

High-level ideas

OK Geometry Plus: Miquel_hyper_proof.pro

File Configure Commands Help

Task Sketch Observe **Project** Report

Miquel theorem

Given is a triangle $\triangle ABC$. Let A' , B' , and C' be arbitrary points on the sides BC , CA , and AB . The three circles through A, B', C' , through A', B, C' , and through A', B', C always meet in a common point.

Observed properties

- Miquel theorem**
Given is a triangle $\triangle ABC$. Let A' , B' , and C' be arbitrary points on the sides BC , CA , and AB . The three circles through A, B', C' , through A', B, C' , and through A', B', C always meet in a common point.
- Strategy of the proof**
Let P be the intersection other than A' of the circles through B, C, A' and through C, A, B' . We shall prove that P lies on the circle through A, B', C' .
- Idea of the proof**
We shall prove that A, C', P, B' are cocyclic, i.e. that $ACPB'$ is a cyclic quadrilateral. To prove this we shall use the theorem:
A quadrilateral is cyclic if and only if its non-adjacent angles are supplementary.
- Theorem**
A quadrilateral $ABCD$ is cyclic if and only if its non-adjacent angles are supplementary.
- Proof** —
(\Rightarrow) Let $ABCD$ be a cyclic quadrilateral. Thus $ABCD$ is inscribed in a circle, let its centre be S . Consider the opposite angles $\angle A$ and $\angle C$. These angles are related to central angles $\angle B'CD$ and $\angle C'AB'$. This

Icon size Emphasize Bleach Highlight Labels Angles Other

1 Miquel theorem

2 Strategy of the proof

3 Idea of the proof

4 Proof

Chaining elements

OK Geometry Plus: Proof1.pro

File Configure Commands Help Development

Task Sketch Observe **Project** Report

Observed properties

1 Task

Given is a circle with centre S and three points, A, B, C on its circumference. Let D be the intersection of the line AB and the bisector of the chord BC . Prove that D lies on the circle through A, S, C .

2 Point D lays on the circle through A, S, C . Why?

3 $\angle CSD = \angle DSB$. Why?

4 $\angle CSD = \angle CAD$. Why?

5 $\angle CSB = 2 \angle CAB$. Why?

Icon size Emphasize Bleach Highlight Labels Angles

1 **Task**

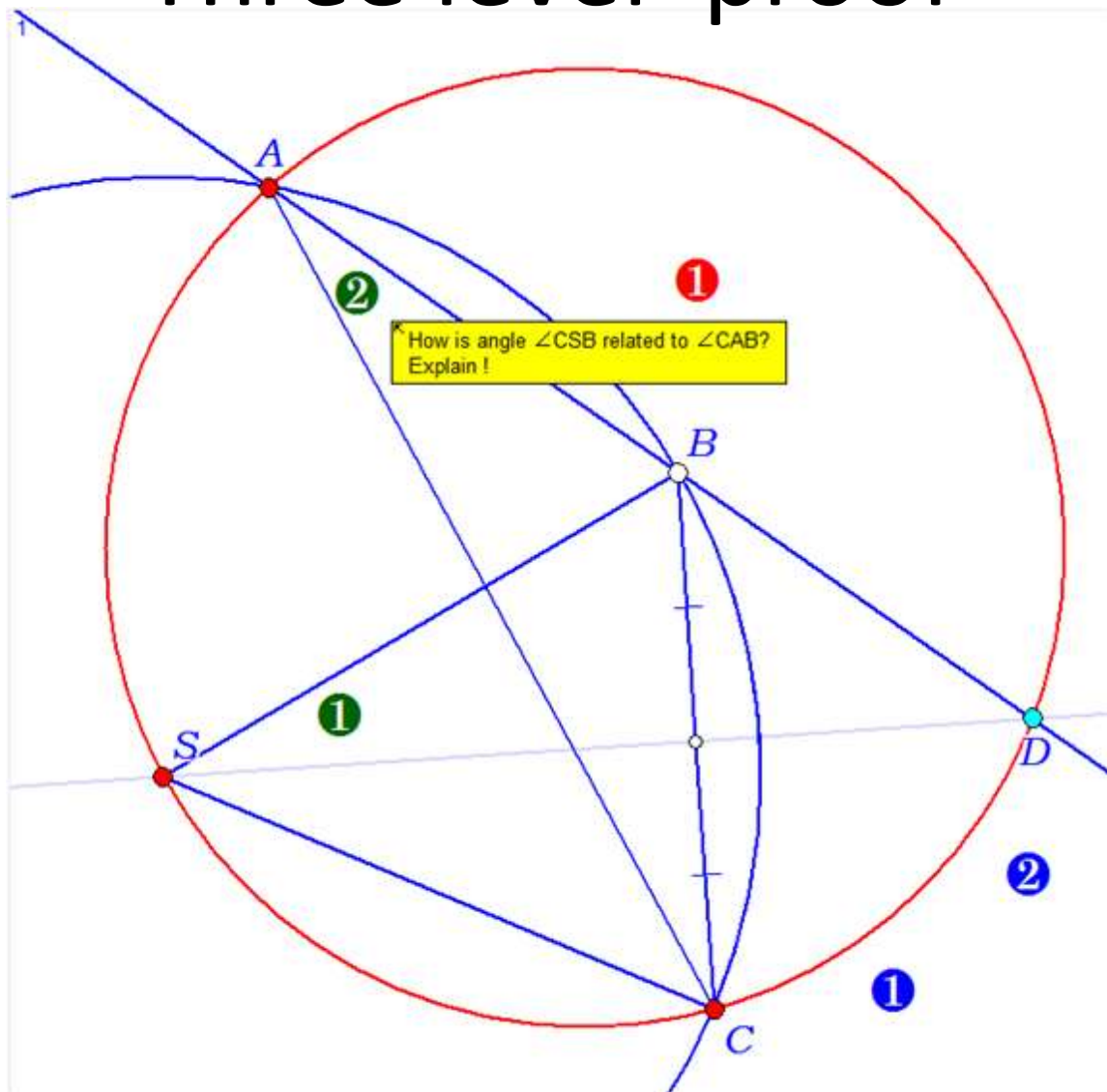
2 Point D lays on the circle through A, S, C . Why?

3 $\angle CSD = \angle DSB$. Why?

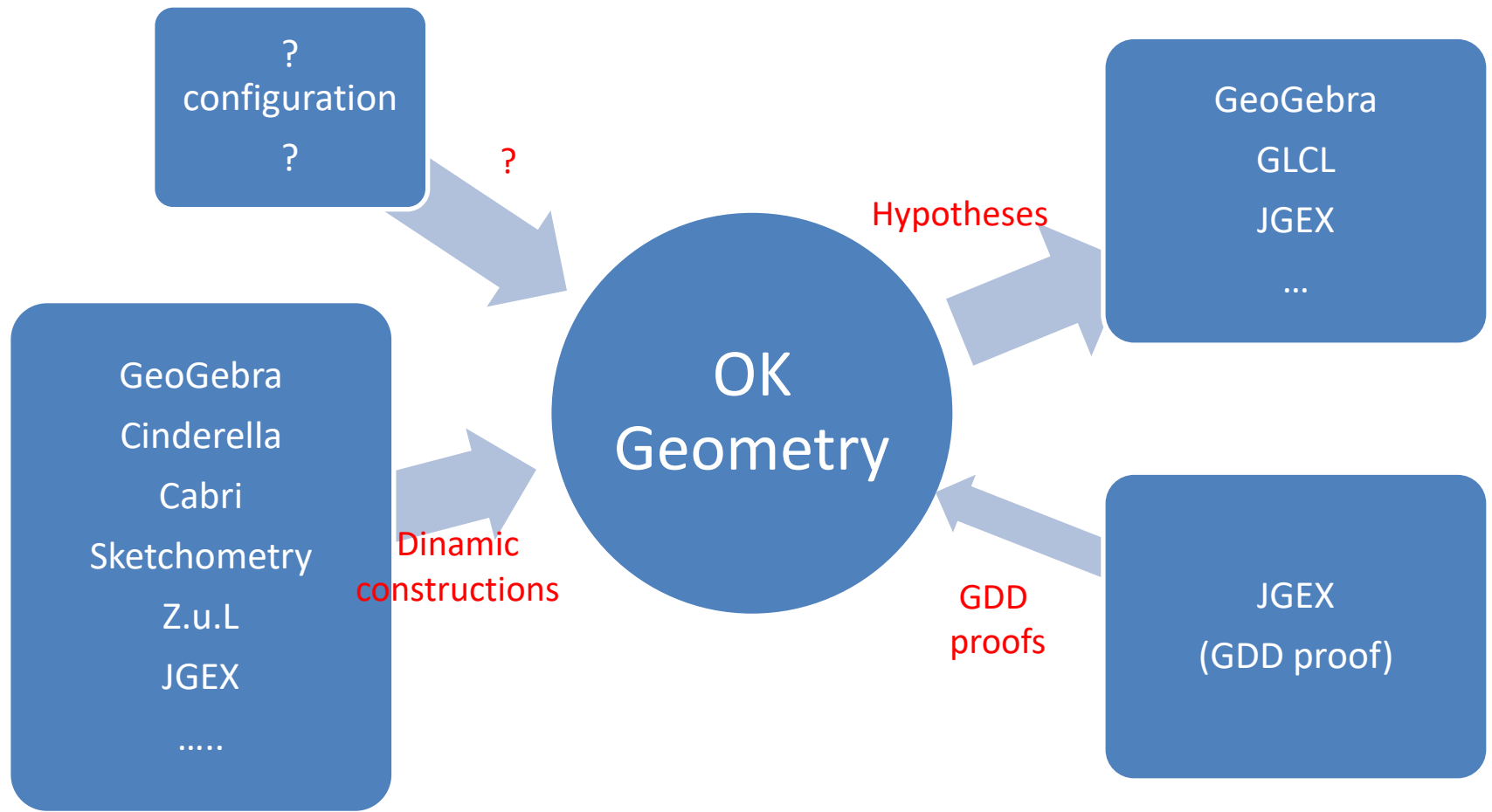
4 $\angle CSD = \angle CAD$. Why?

5 $\angle CSB = 2 \angle CAB$. Why?

'Three level' proof



Observation and (A)DG tools



Thanks