OK Geometry - observing dynamic constructions

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You can downolad OK Geometry at

www.ok-geometry.com

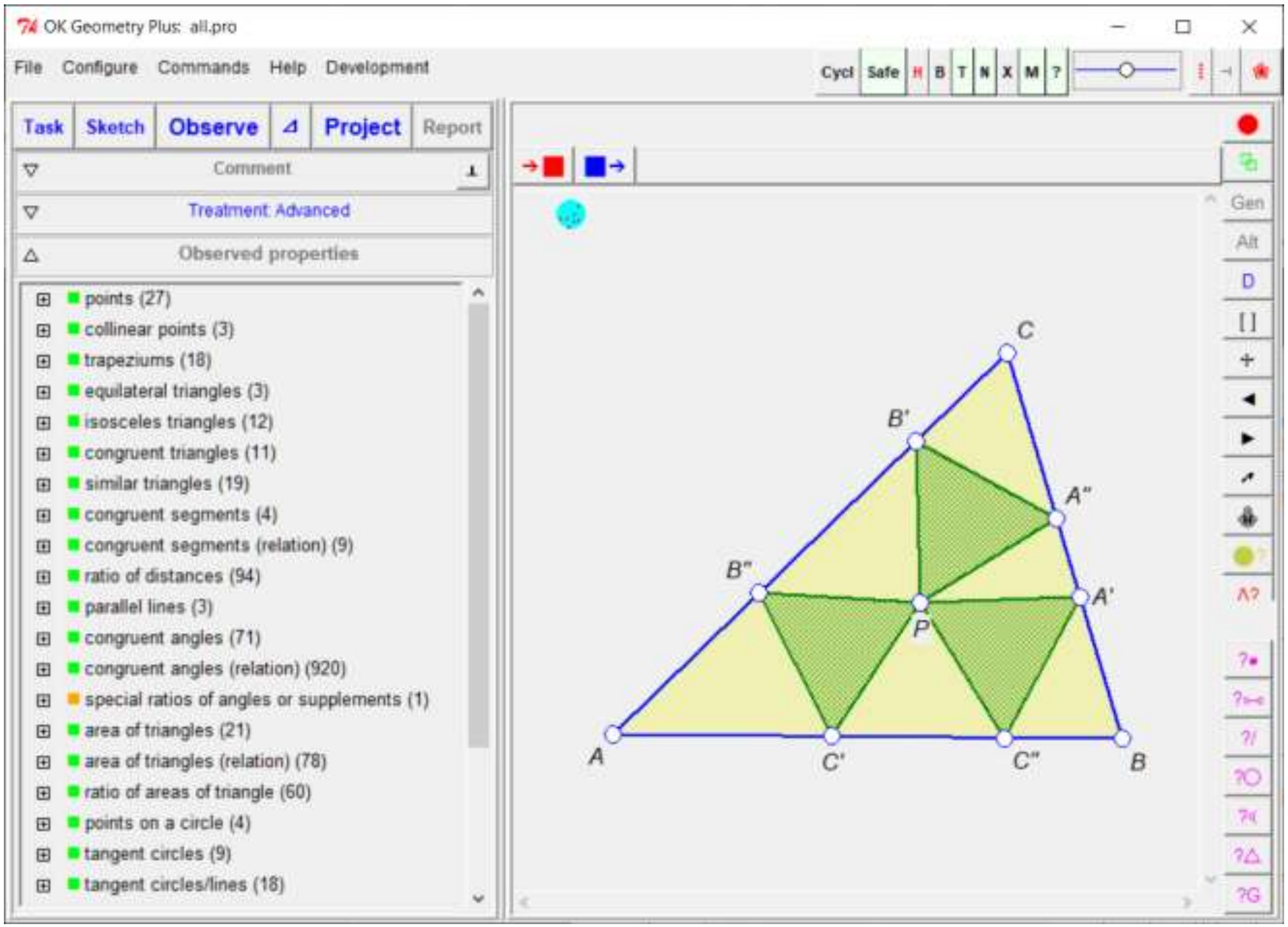
Please download the version 19.4.4 .

Hereby shown material (and more) is in

ADG 2023 section

Unzip and launch OKGeometry_19_4.exe

OK Geometry - A tool for observing dynamic constructions



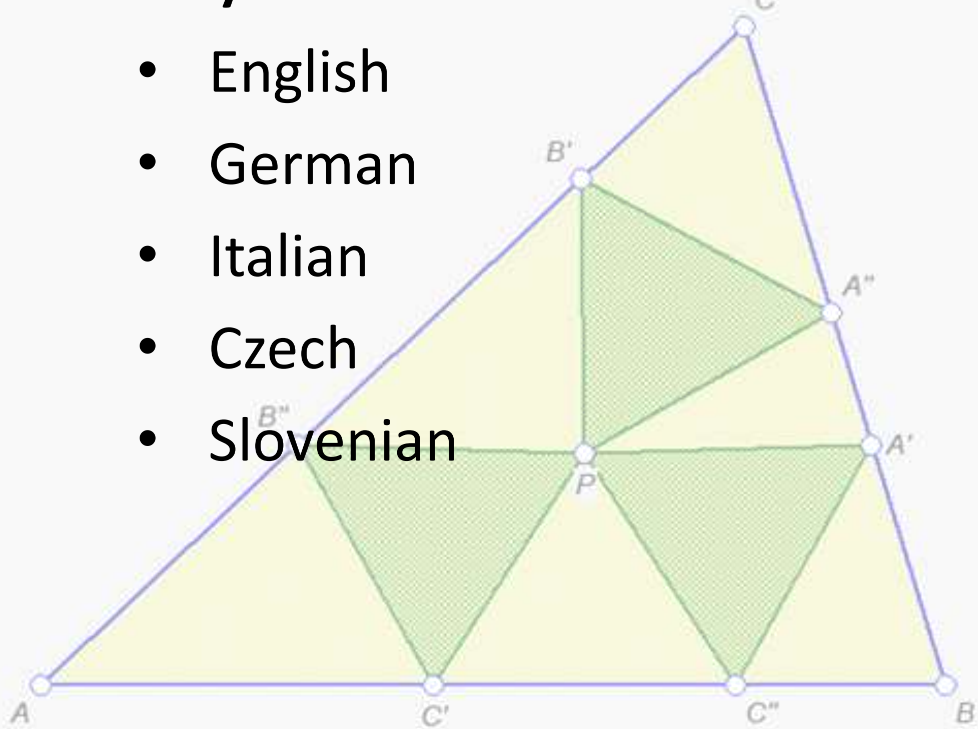
OKG - A tool for observing dynamic constructions

3 working modes

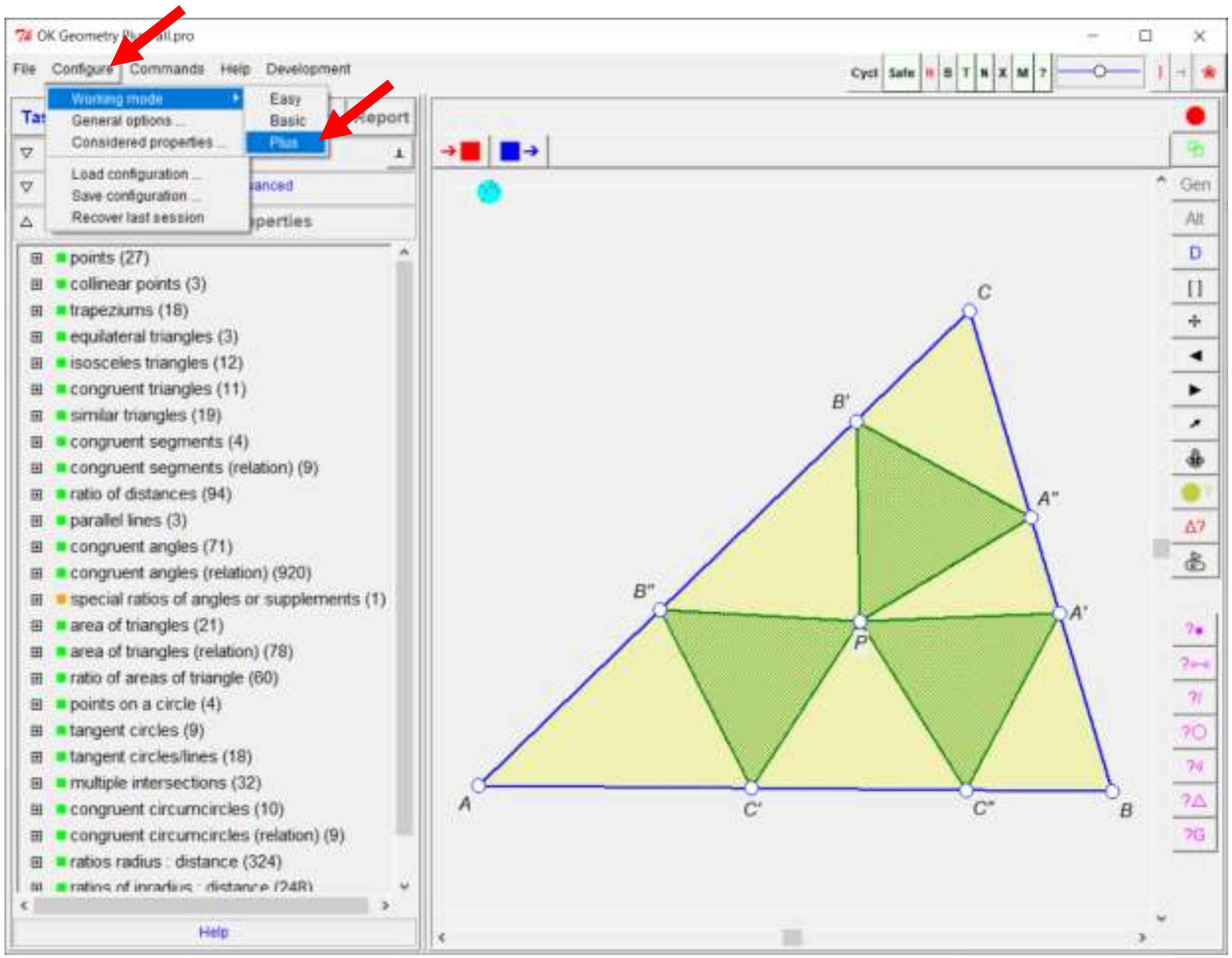
- **Easy** (lower secondary level)
- **Basic** (upper secondary level)
- **Plus**

Easy and Basic level available in

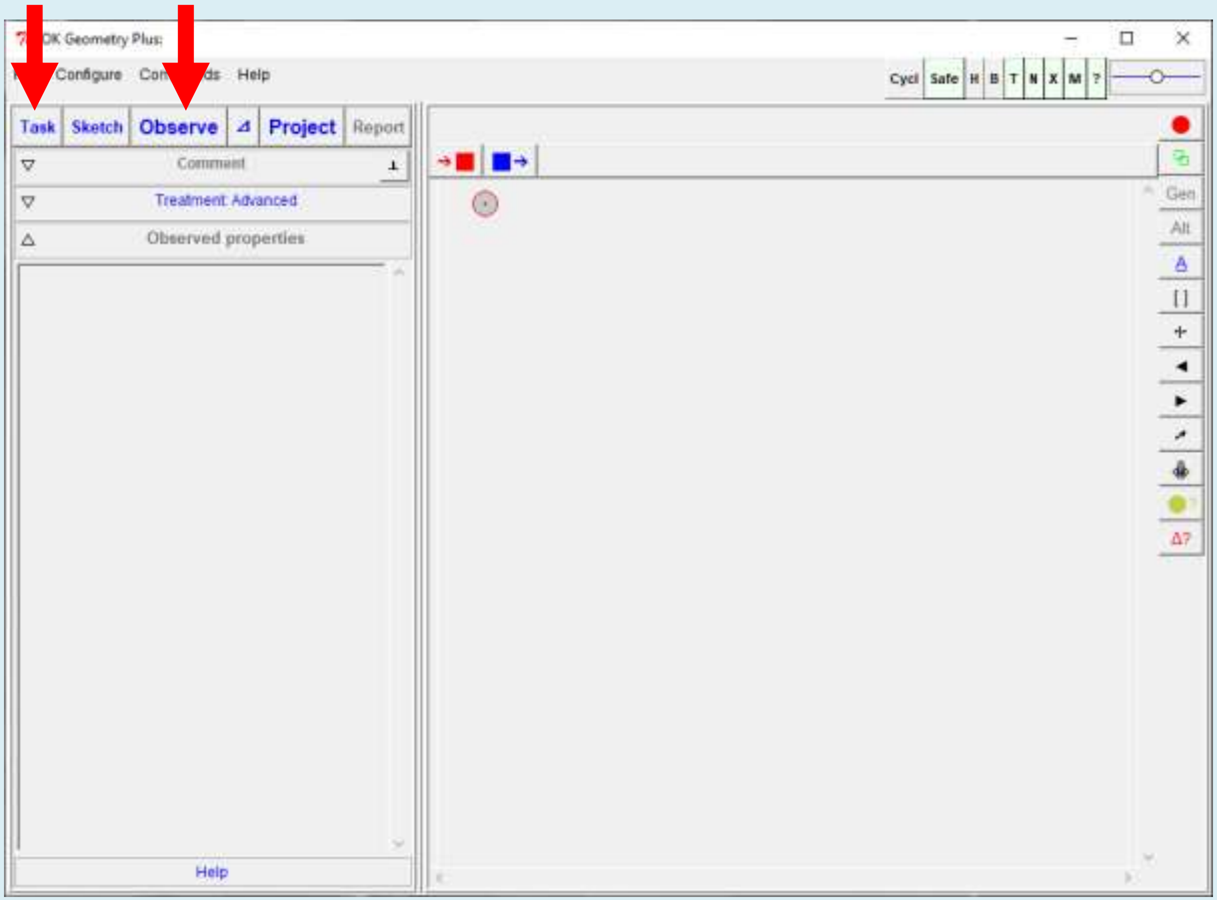
- English
- German
- Italian
- Czech
- Slovenian



OKG - A tool for observing dynamic constructions

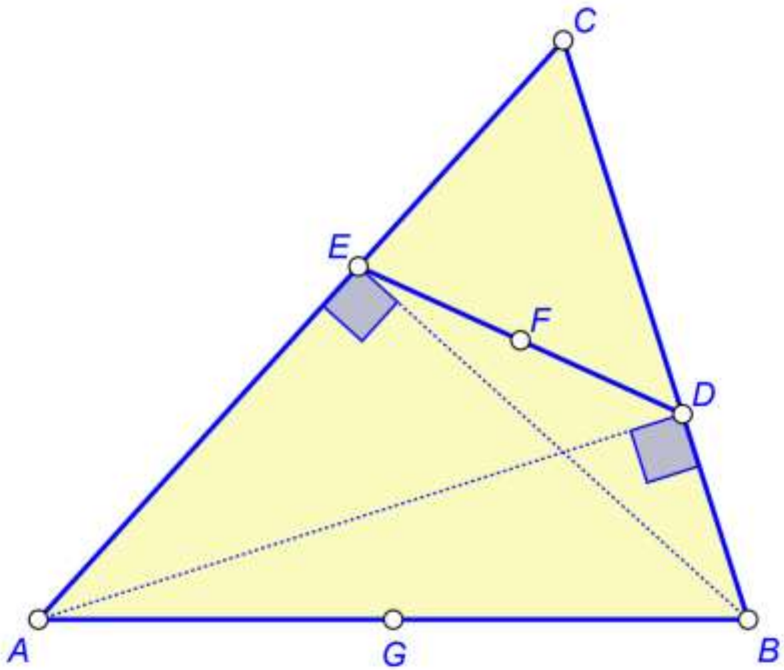


Simple observation of dynamic constructions



- Observe properties of a dynamic construction
- ‘Restricted’ observation
- Observing algebraic relations

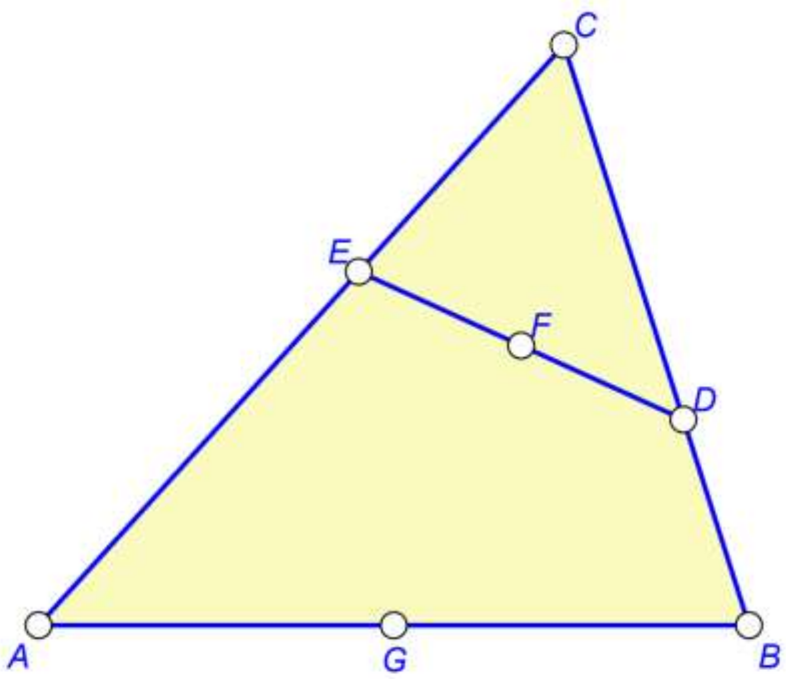
Observing imported constructions



- ABC - a triangle
- D – base of A-altitude
- E – base of B-altitude
- F – midpoint of DE
- G – midpoint of AB

Observe the properties of this configuration.

Observing imported constructions



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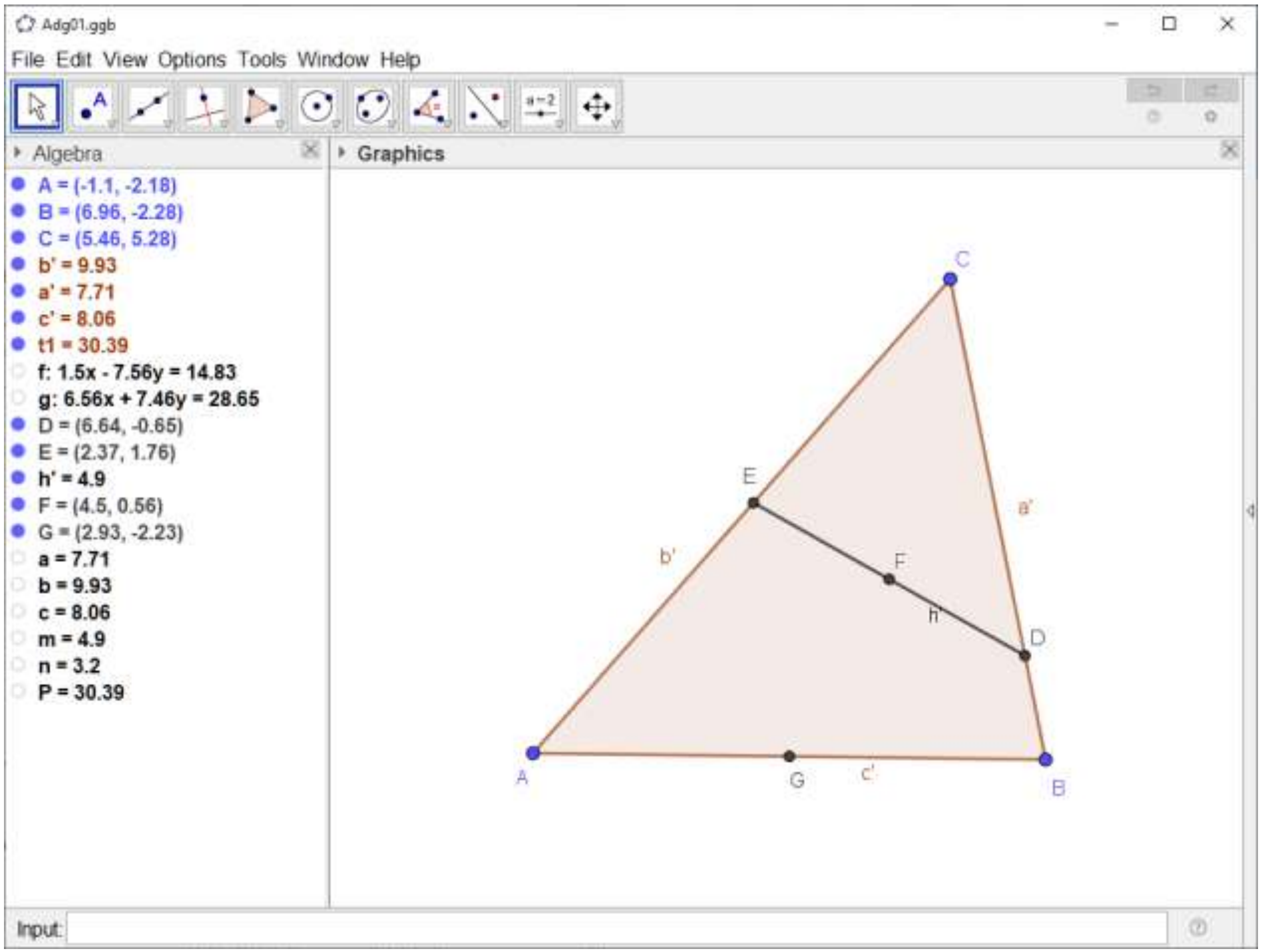
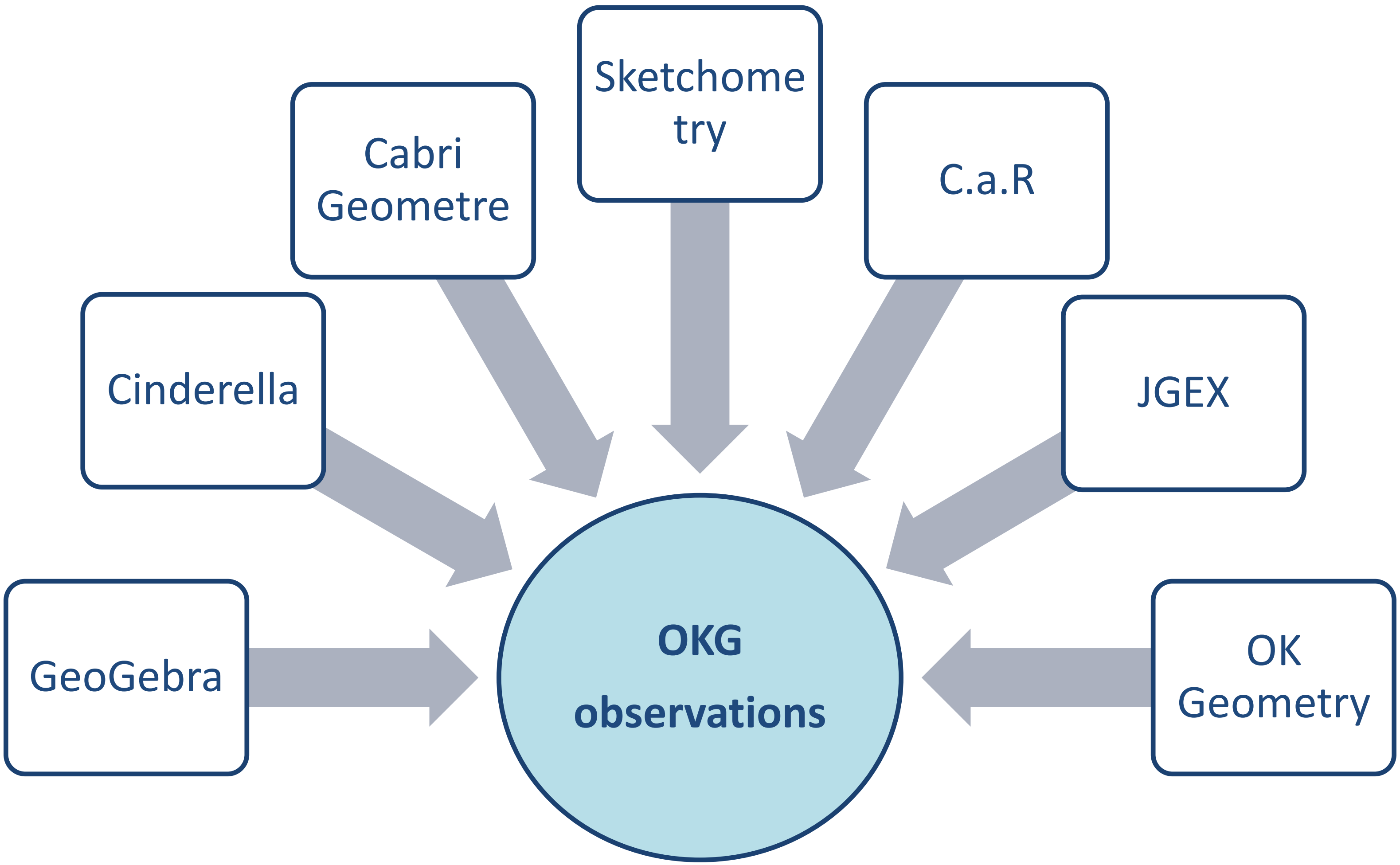
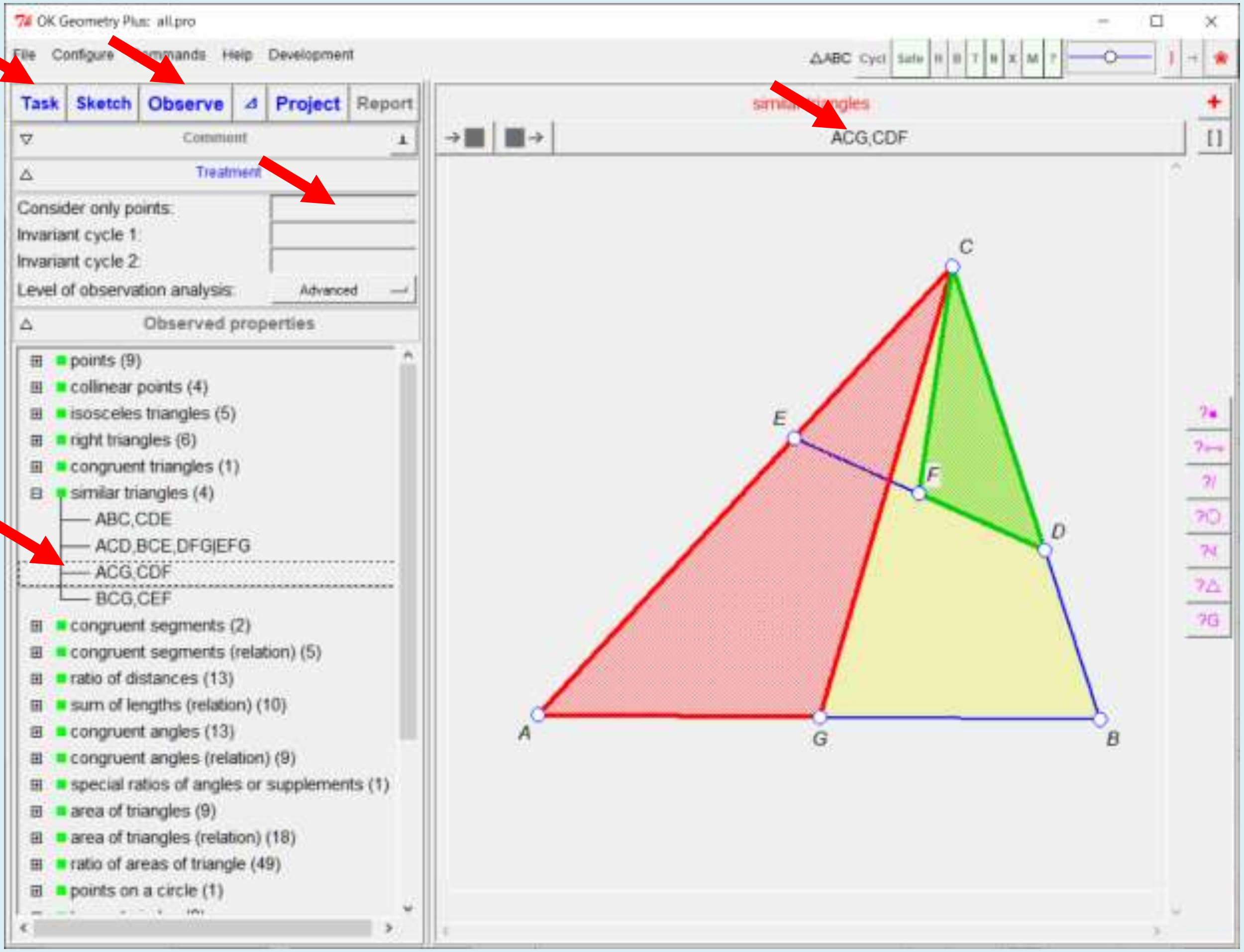

Adg01.ggb
File Edit View Options Tools Window Help
Algebra
Graphics
A = (-1.1, -2.18)
B = (6.96, -2.28)
C = (5.46, 5.28)
b' = 9.93
a' = 7.71
c' = 8.06
t1 = 30.39
f: 1.5x - 7.56y = 14.83
g: 6.56x + 7.46y = 28.65
D = (6.64, -0.65)
E = (2.37, 1.76)
h' = 4.9
F = (4.5, 0.56)
G = (2.93, -2.23)
a = 7.71
b = 9.93
c = 8.06
m = 4.9
n = 3.2
P = 30.39
C
E
a'
b'
F
h'
D
A
G
c'
B
Input:

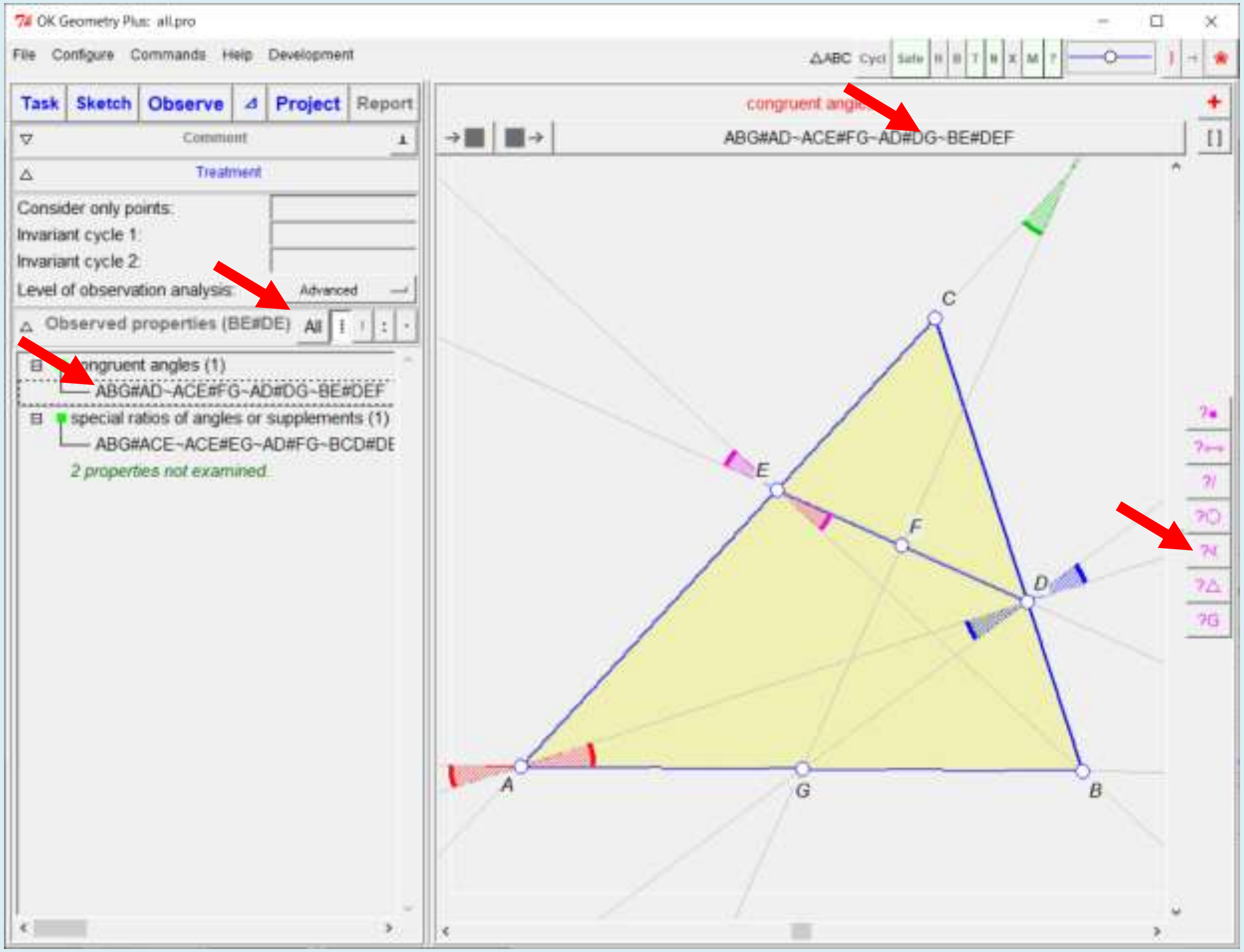
Importing constructions from DGS



Observing imported constructions



Observing imported constructions



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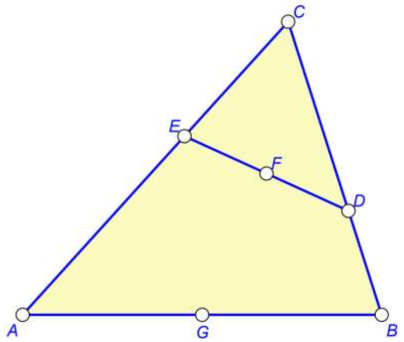
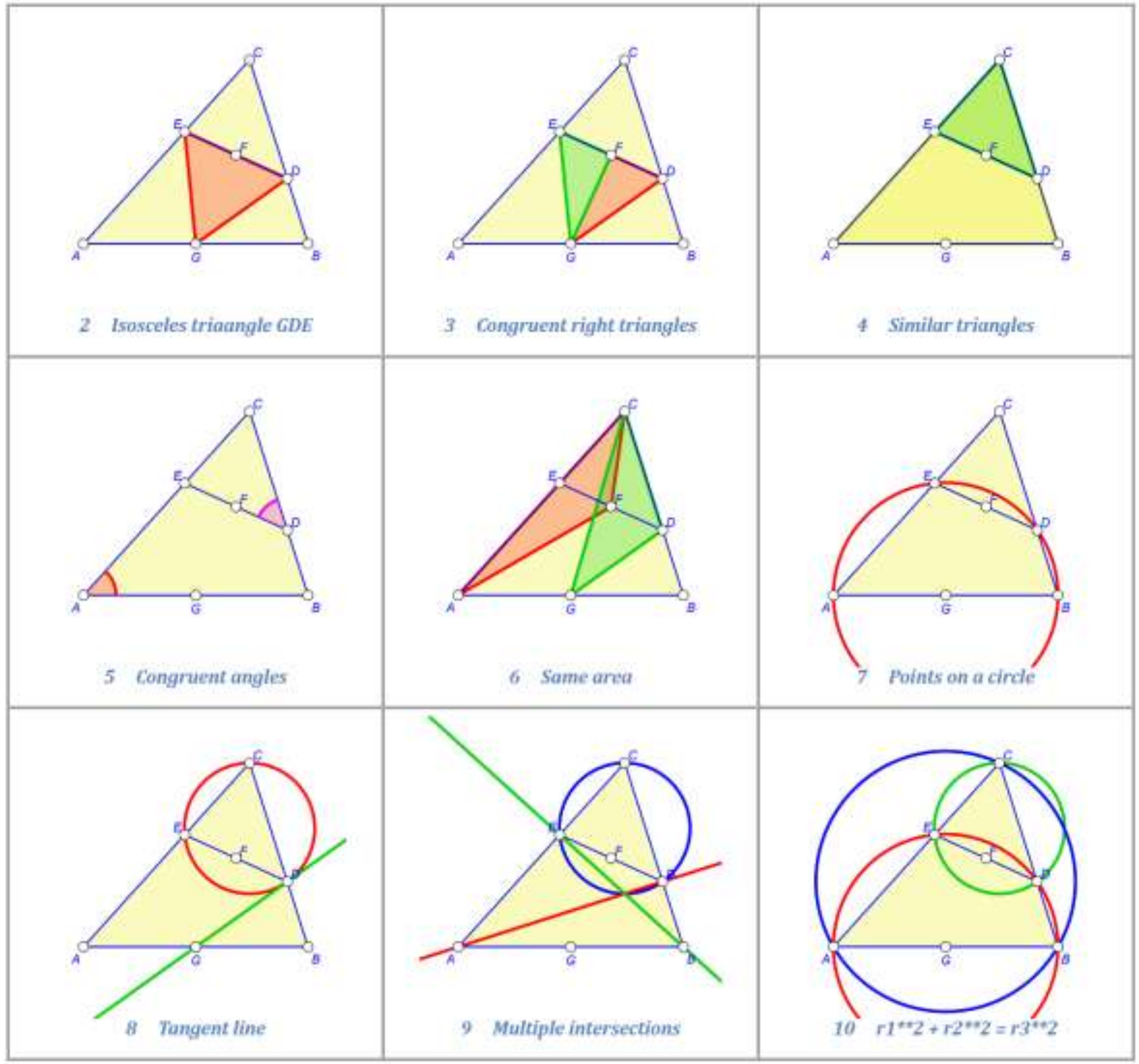
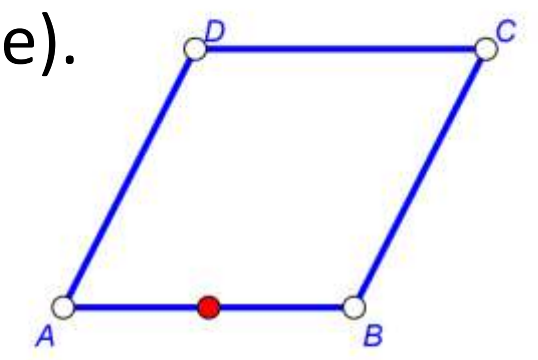




Understanding properties

- OKG considers the displayed objects and objects passing through **labelled points**.
- Advice: label 3-12 relevant points.
- OKG considers only angles between lines (angle ≡ supplementary angle).
- OKG ignores trivial congruences of angles between lines.



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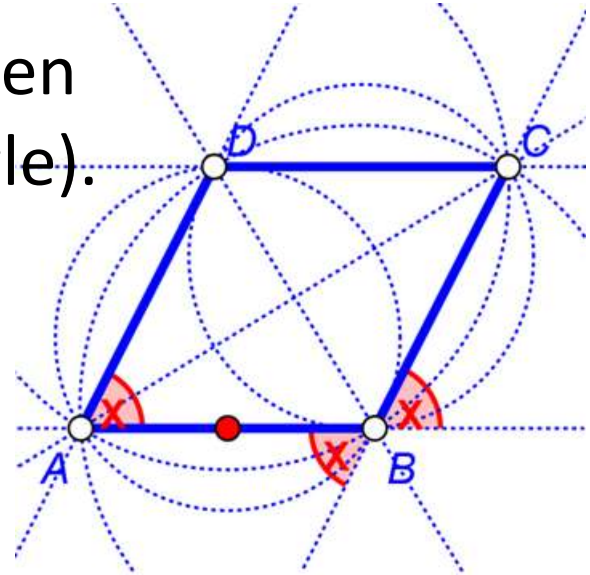
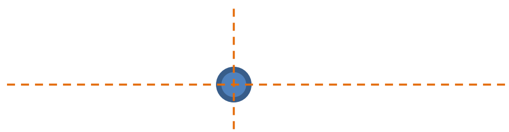


Models of geometry

Static model

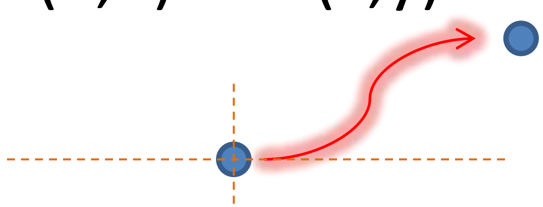
Free point A(3,5)



Dynamic model

Free point

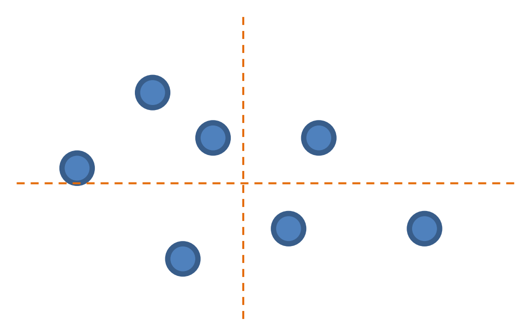
A(3,5) -> A(x,y)



Stochastic dynamic model

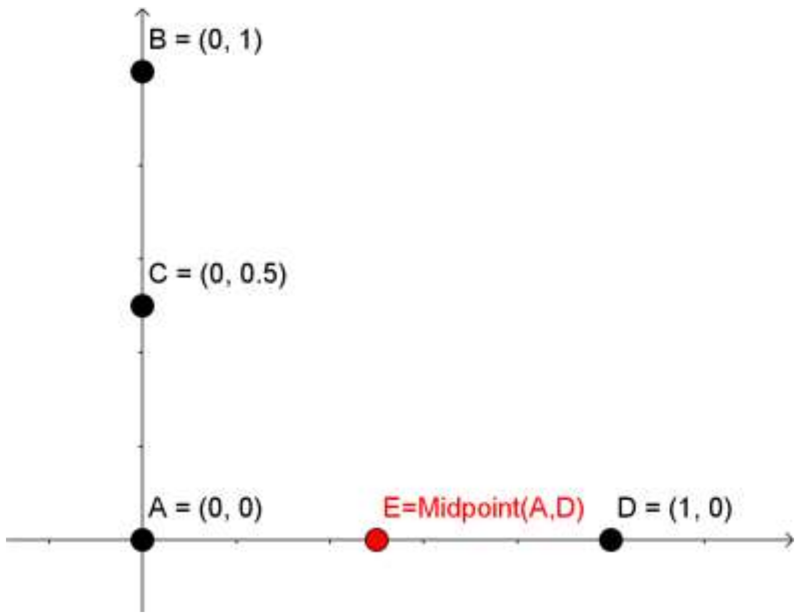
Free point A(3,5) ->

$\{(x_1,y_1), (x_2,y_2), \ldots ,(x_k,y_k) \}$

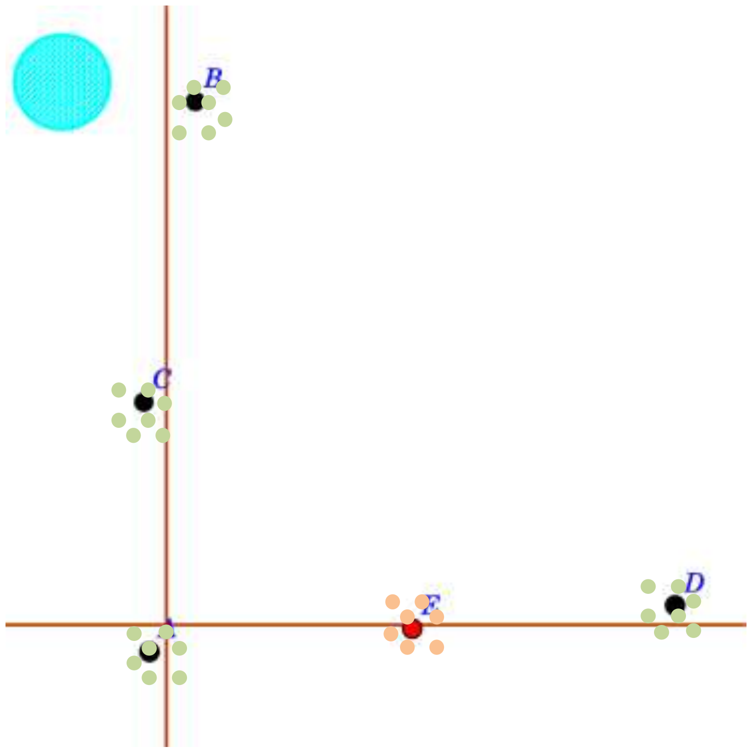


Randomisation of constructions

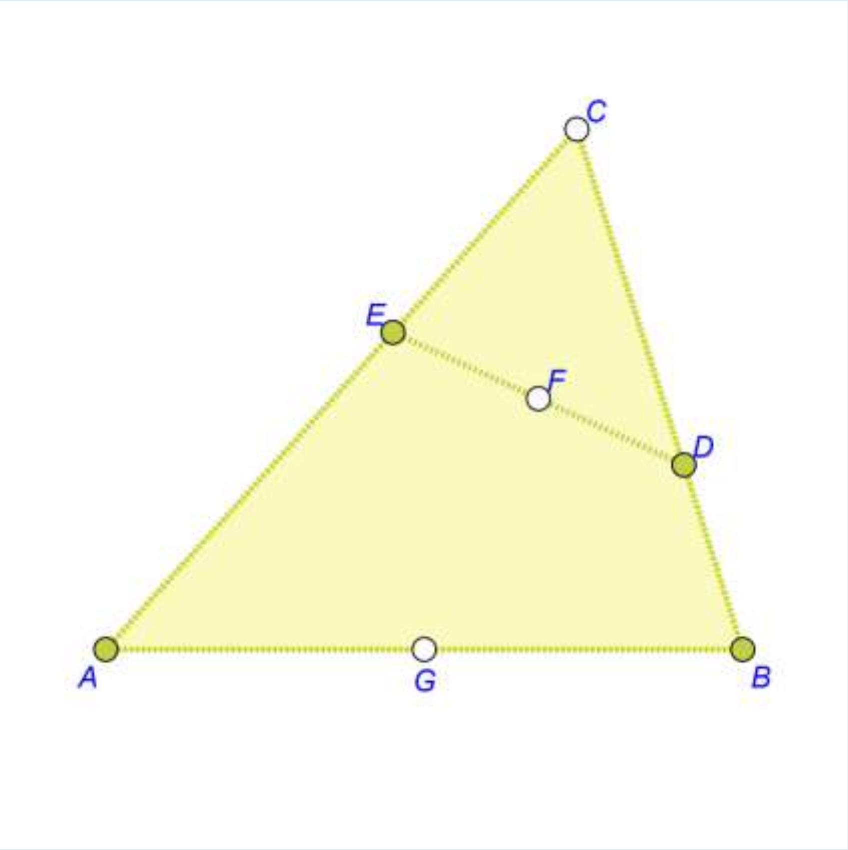
GeoGebra



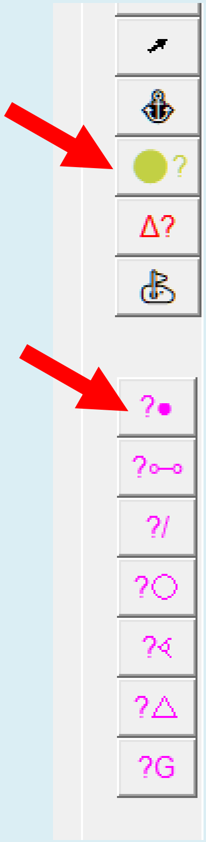
Instance of construction



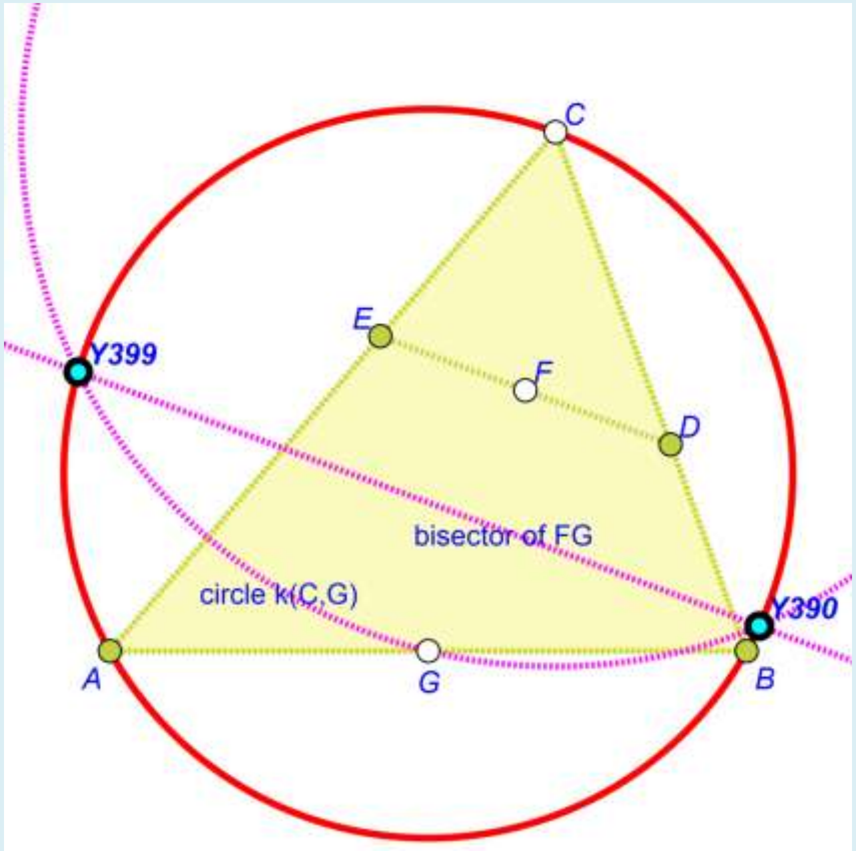
Advanced observation

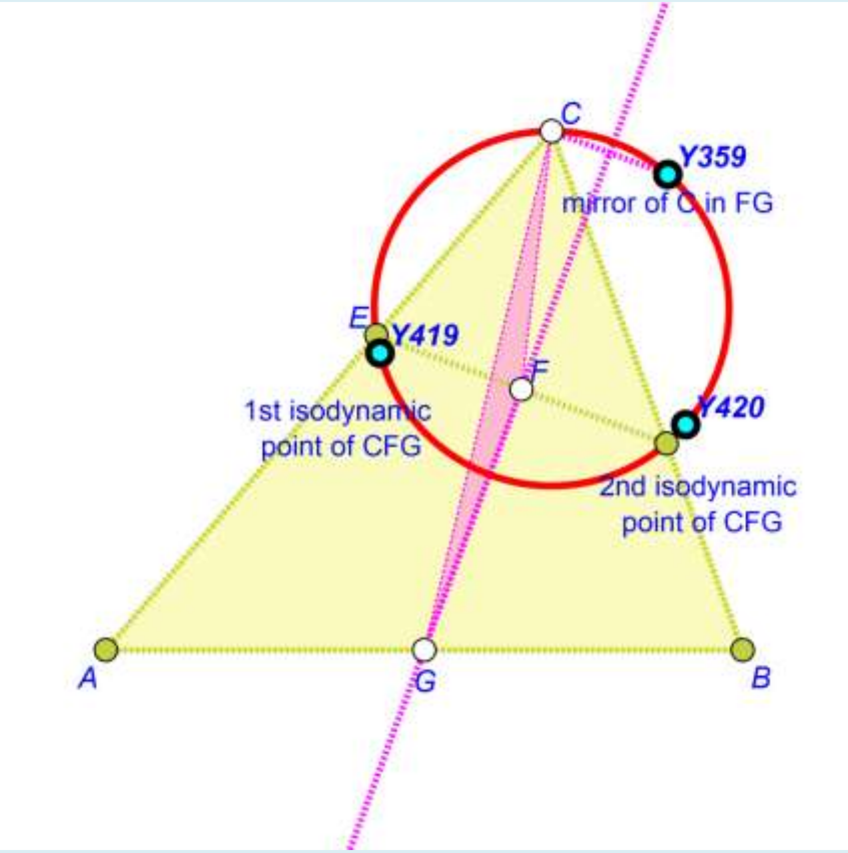


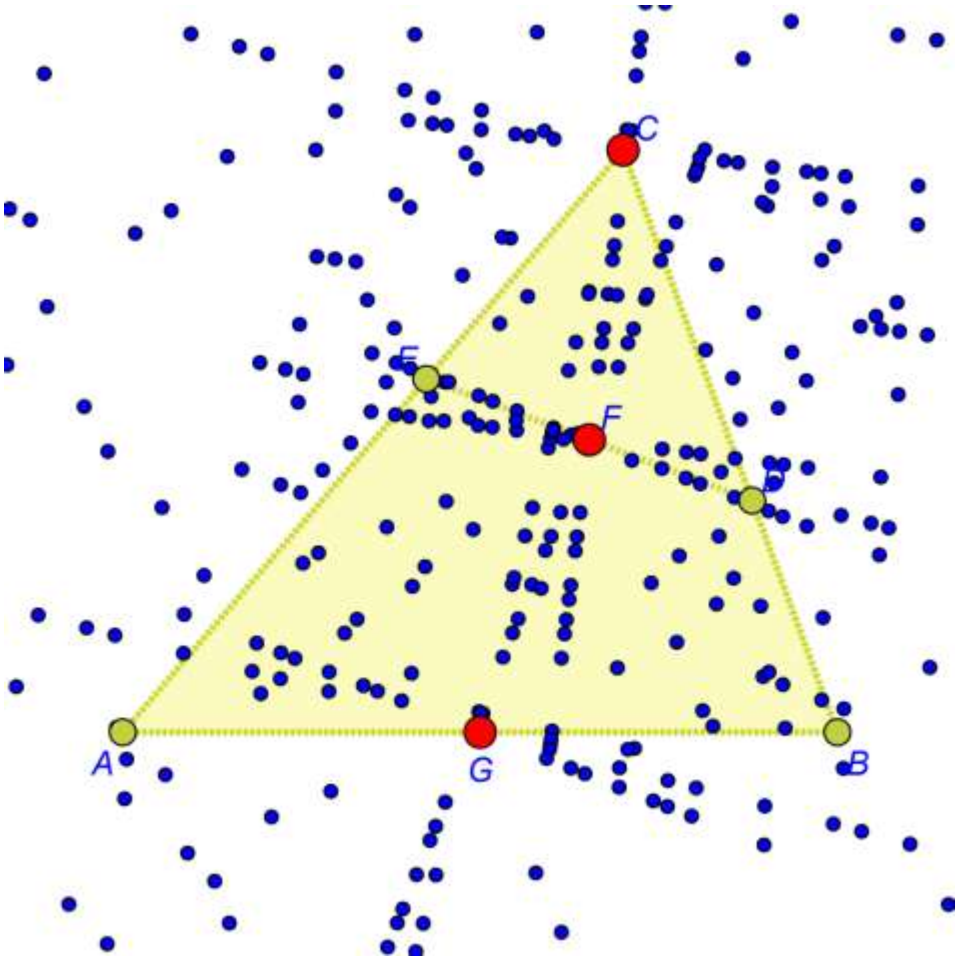
- How to **construct** the triangle ABC from known positions of points C, F, G.



Advanced observation

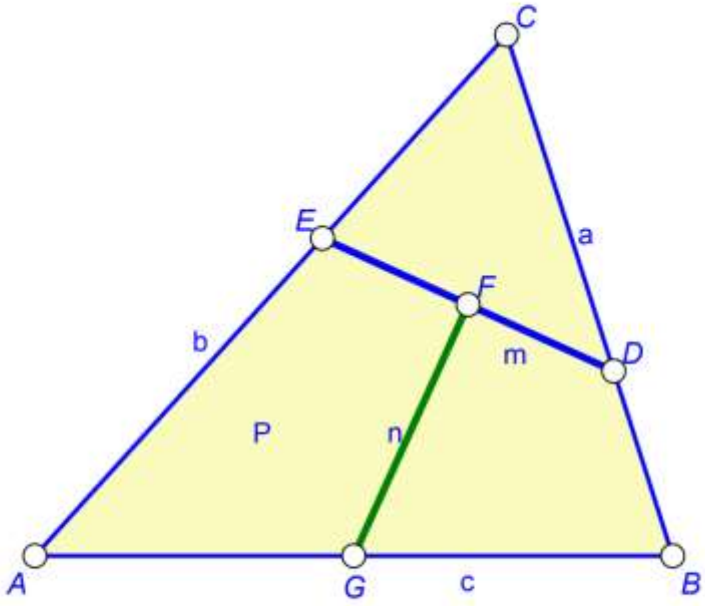






C
E
F
D
A
G
B

Observing algebraic relations



P = Area(A,B,C)

a = Distance(B,C)

b = Distance(C,A)

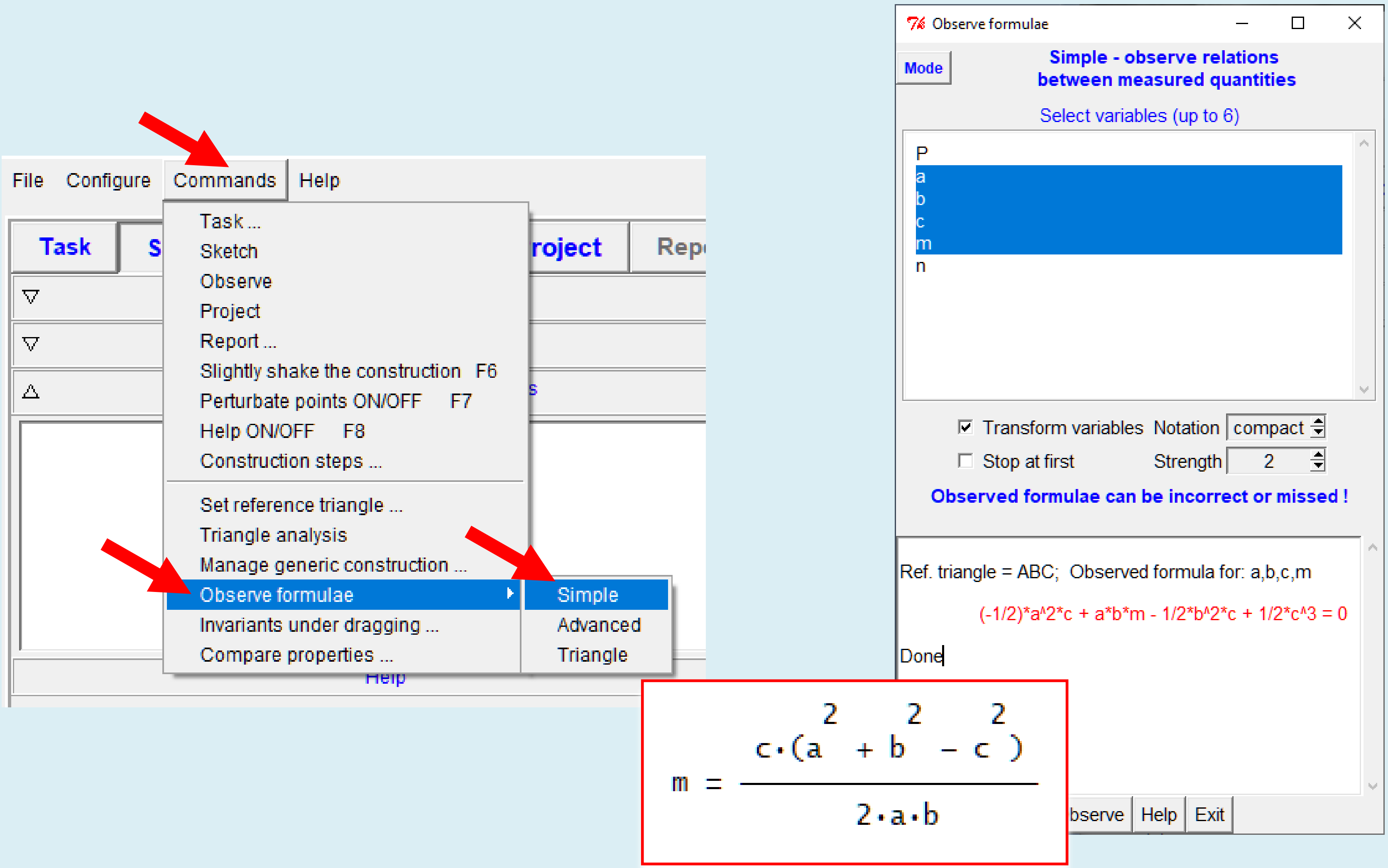
c = Distance(A,B)

m = Distance(D,E)

n = Distance(F,G)

Note. Use explicit measurements.

Observing algebraic relations



Observing algebraic relations

- Consider several instances of a construction to obtain several instances of parameters (x_1, x_2, …x_k).
- Solve the a system of linear equations

$$\sum_{n_1+n_2+\cdots+n_k<r} \alpha_{n_1.n_2,\ldots,n_k} {x_1}^{n_1} {x_2}^{n_2} \ldots {x_k}^{n_k} = 0$$

- Technical problems…

The principle of simple observation

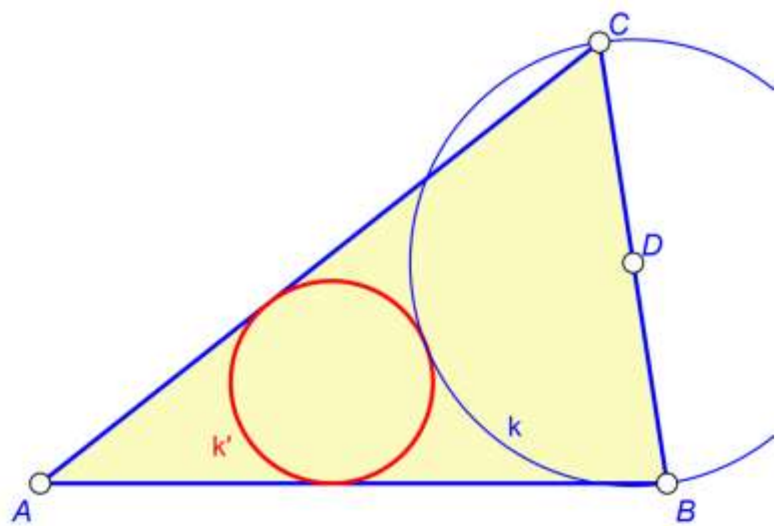
A construction

Random realisations
(rand_fig1, rand_fig2, rand_fig3,....)

Common simple numer. properties
(eg. AB=AC)

Textual elaboration for properties
(eg. ABC is isosceles)

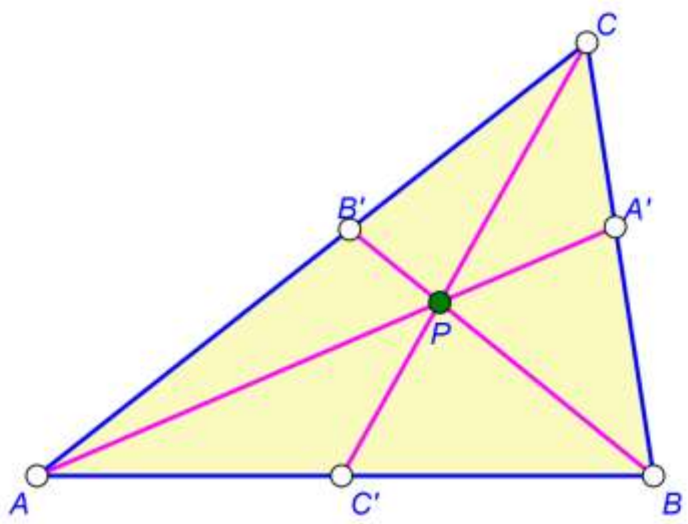
A ‘difficult’ object



- ABC – an acute triangle
- *k* = *k*(D,B) – circle with diameter BC
- *k*’ – a circle inscribed in the ‘triangle’ bound by AB, *k* and CB.

- Analyse the circle k’.

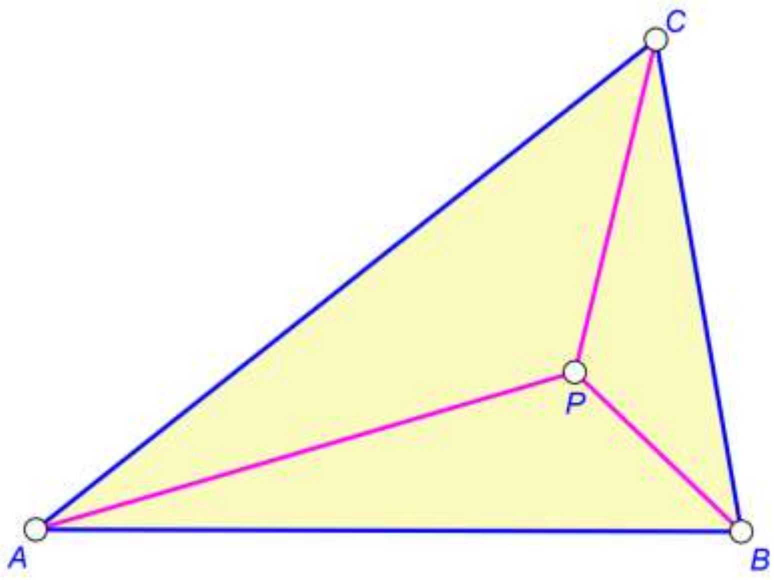
An ‘implicit’ object



- ABC – a triangle
- P – a point
- AA’, BB’, CC’ – Cevian lines of P in ABC.
- **AA’ ≡ BB’ ≡ CC’**

Investigate!

An optimisation problem

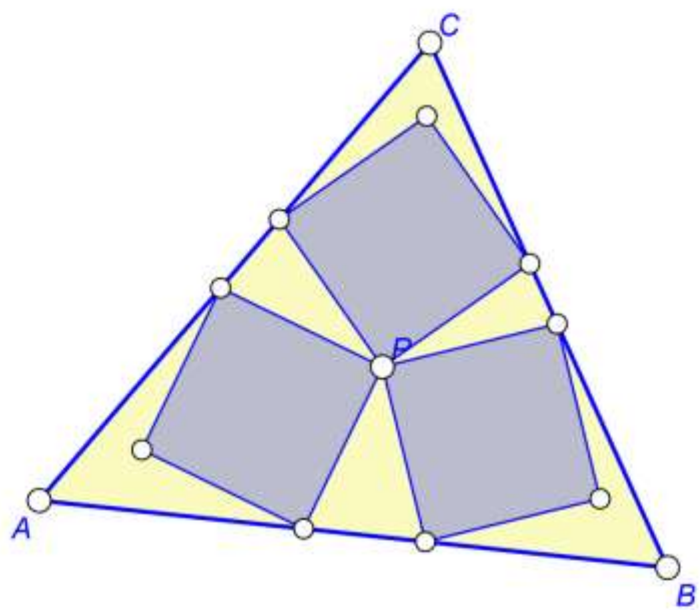


ABC – **reference triangle**

P – point on plane that **minimises** **|AP|+|BP|+|CP|**.

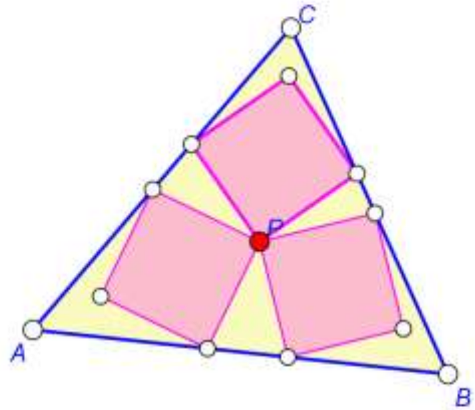
Analyse the position of such a point P.

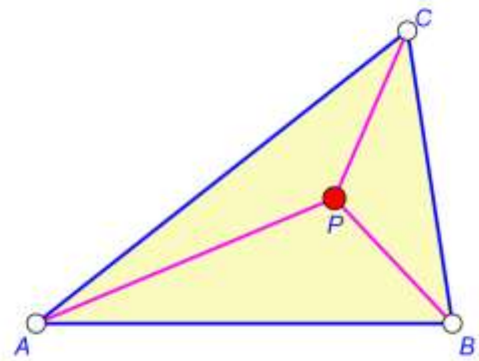
A nice problem

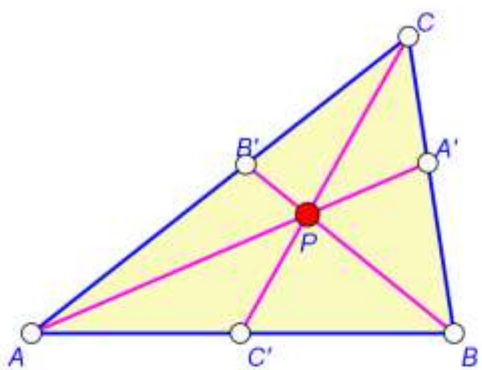


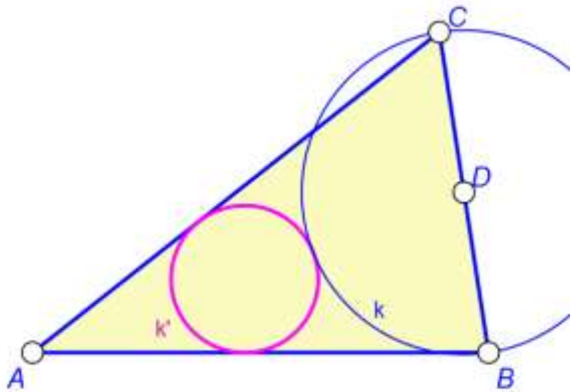
How to inscribe 3 congruent squares into a given triangle ABC as shown in the figure?

How to observe?





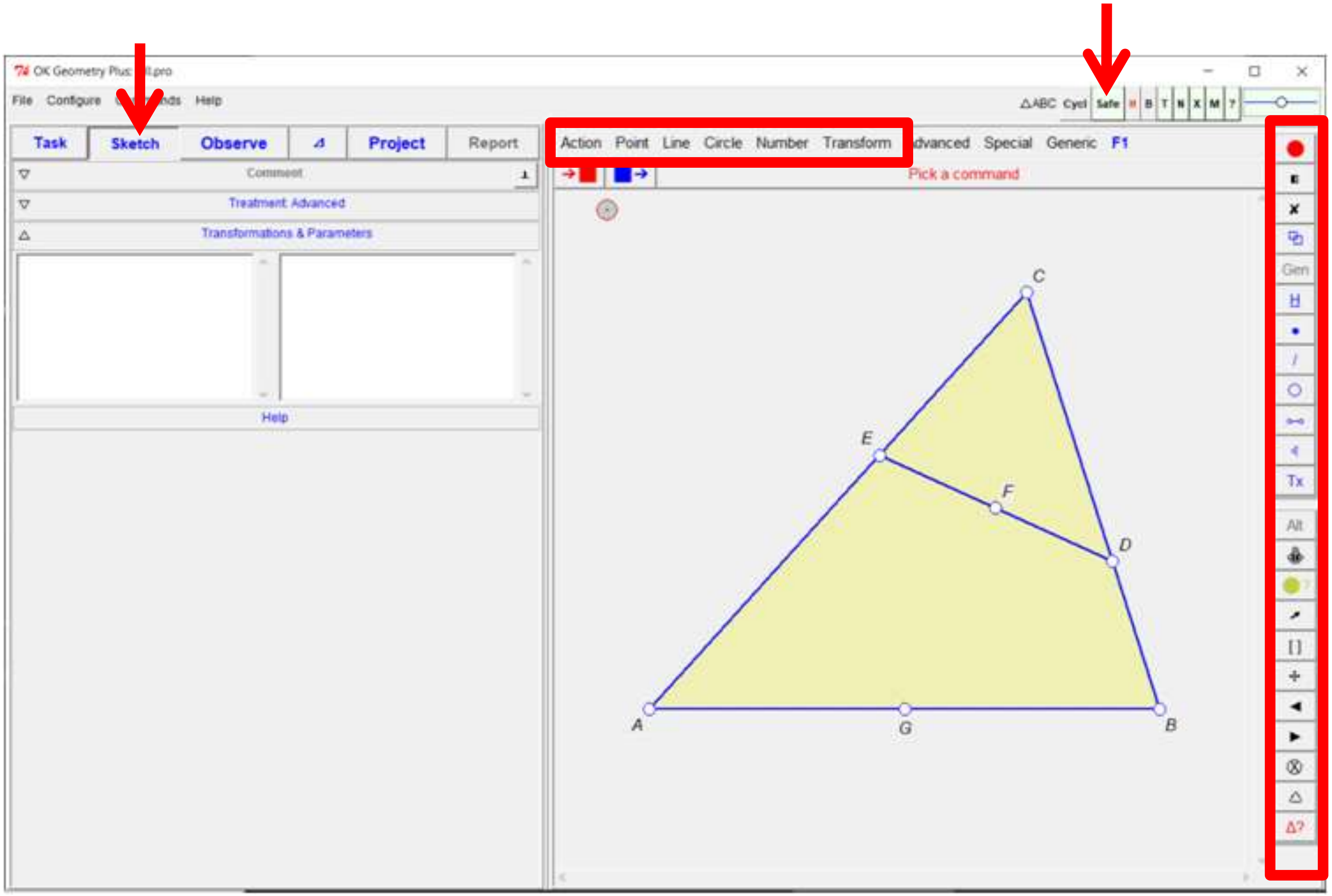




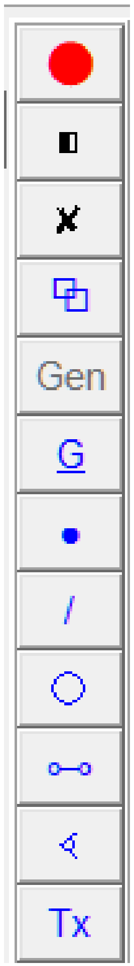
OKG Sketch Editor

- Configuration vs. construction
- OKG observation requires (several) ‘exact’ configurations.
- Sketch Editor creates
  - Constructions
  - Difficult objects
  - Implicit constructions (configurations)
  - Configurations by optimisation

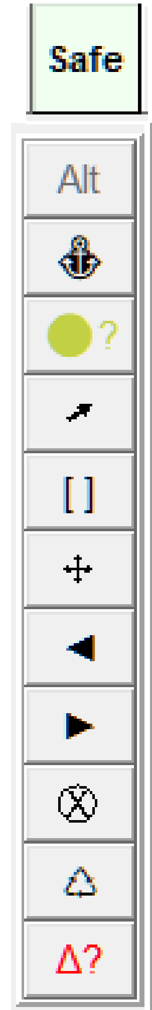
OKG Sketch Editor



OKG Sketch Editor – common buttons



Shape objects
Hide objects
Delete objects
(Scenes,…)
(Generic view)
Label points
Point, Intersection, Midpoint
Line, Line 2 obj
Circle, Circle 3 obj, …
Segment, Perp.seg., Polyline
Angle, Various decorations
Text

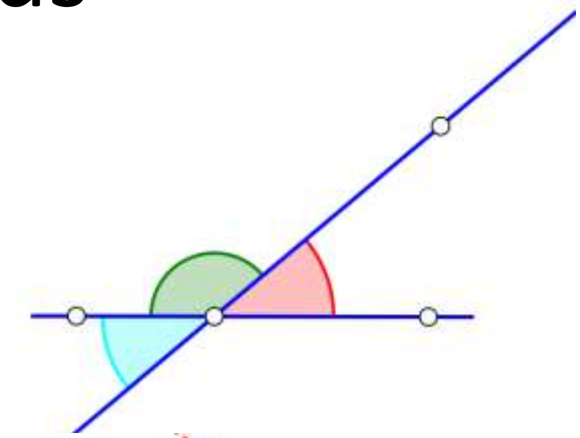


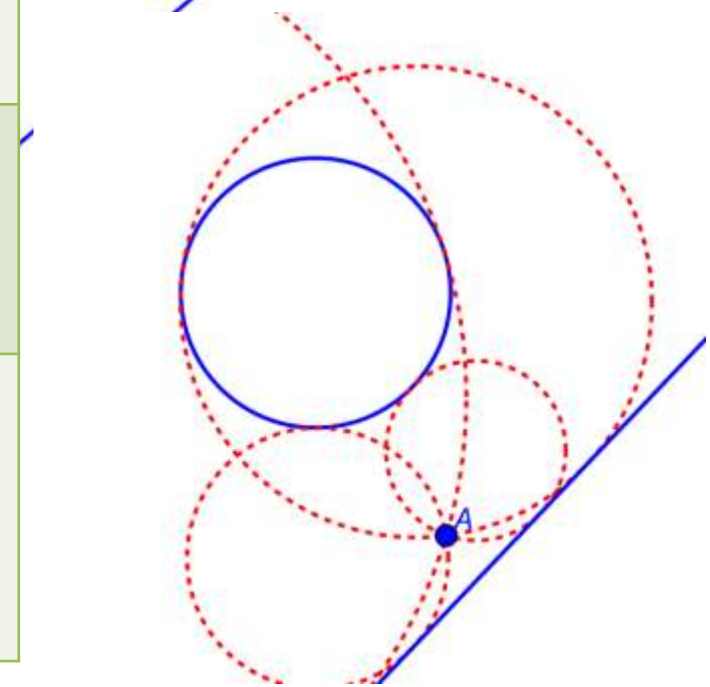
Safe objects
Alternative objects
Anchor
(Mark Unknown)
Drag point
Zoom view …
Move view
Undo
Redo
Redefine
(Declare cyclic)
(Triangle analysis)

F8 – Help ON/OFF

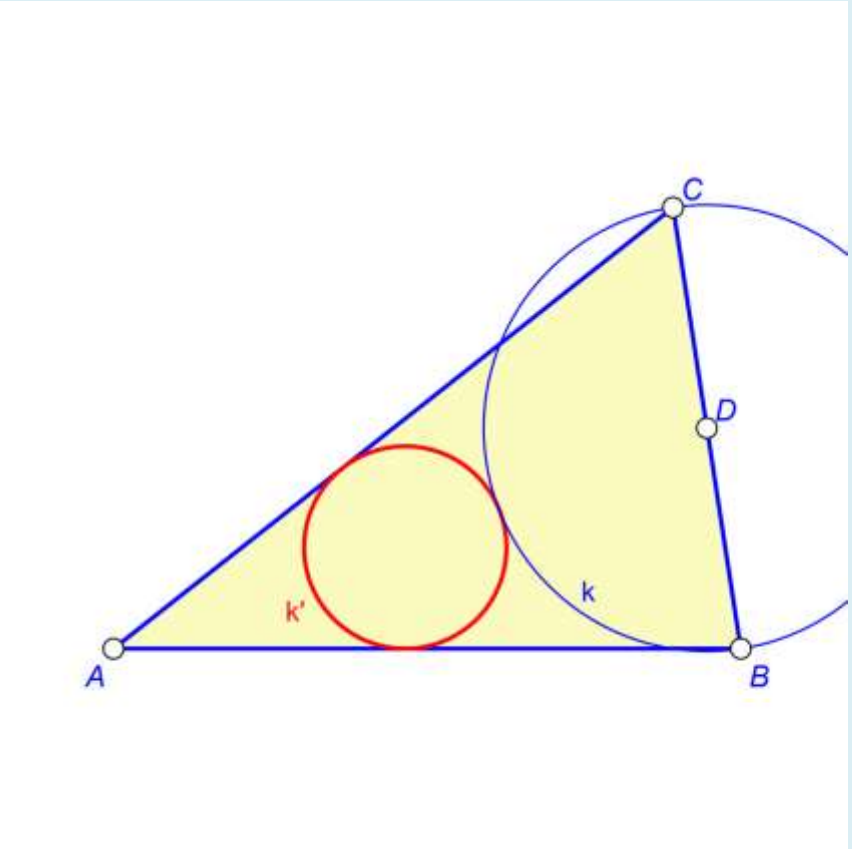
OKG Sketch Editor – special commands

	Safe ON	When necessary, segments are treated as lines, arcs as circles.
Alt	**Alt** (try mouse scroll)	Press repeatedly for alternative solutions.
	Anchor (otrymouse scroll)	Press repeatedly for different ways of representation of objects,
%	**Line 2 objects** + **Alt** (try mouse scroll)	Line defined by 2 objects in terms of ‘passing through’, ’is parallel’, ’is tangent’, ’is radical axis’.
③	**Circle 3 objects** + **Alt** (try mouse scroll)	Circle defined with 3 objects in terms of ‘passing through’, ‘is tangent’.





A ‘difficult’ circle



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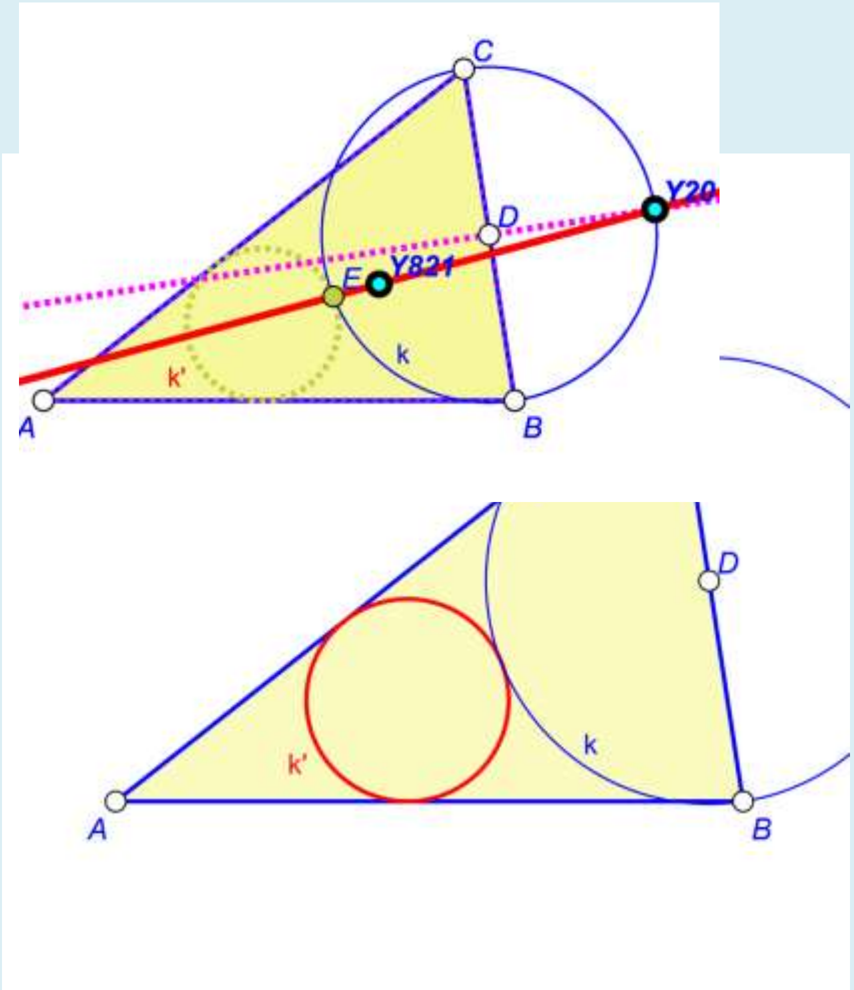


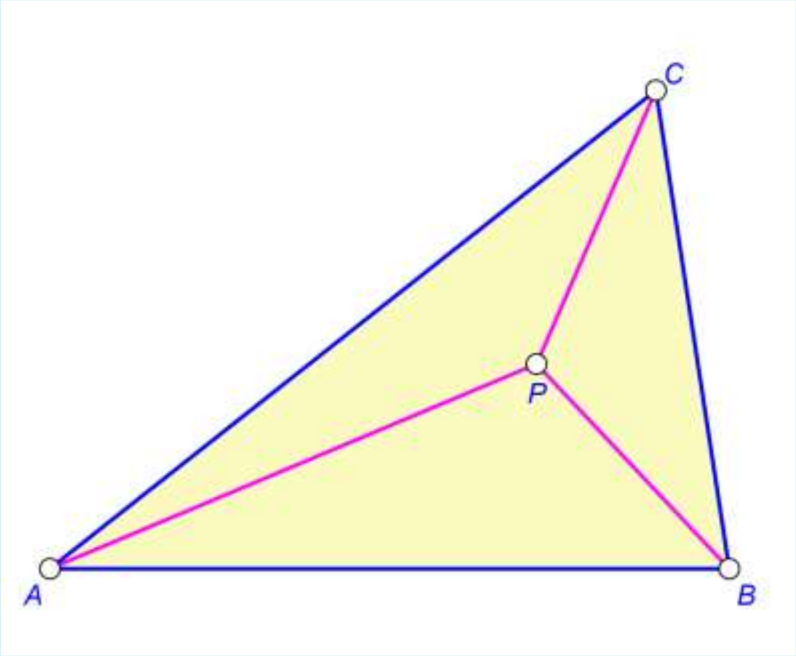
1*ra*a + ra*b + ra*c + 1/2*a^2 - 1/2*b^2 + b*c - 1/2*c^2 - S = 0

1*ra*a + ra*b + ra*c + 1/2*a^2 - a*ri - 1/2*b^2 + b*c - b*ri - 1/2*c^2 - c*ri = 0

1*ra*a*(r*cos(A)) + ra*b*(r*cos(A)) + ra*c*(r*cos(A)) + 1/4*a^3 + 1/2*a^2*(r*cos(A)) - 1/4*a*b^2 - 1/4*a*c^2 - 1/2*b^2*(r*cos(A)) + b*c*(r*cos(A)) - 1/2*c^2*(r*cos(A)) = 0

(-1/2)*ra^2*(r*cos(A)) - 1/4*ra^2*(r*cos(B)) - 1/4*ra^2*(r*cos(C)) + 1/4*ra^2*ri + ra*(r*cos(A))*ri + 1/2*ra*(r*cos(B))*ri + 1/2*ra*(r*cos(C))*ri - 1/2*ra*ri^2 - 1/2*(r*cos(A))*ri^2 = 0

An optimisation problem



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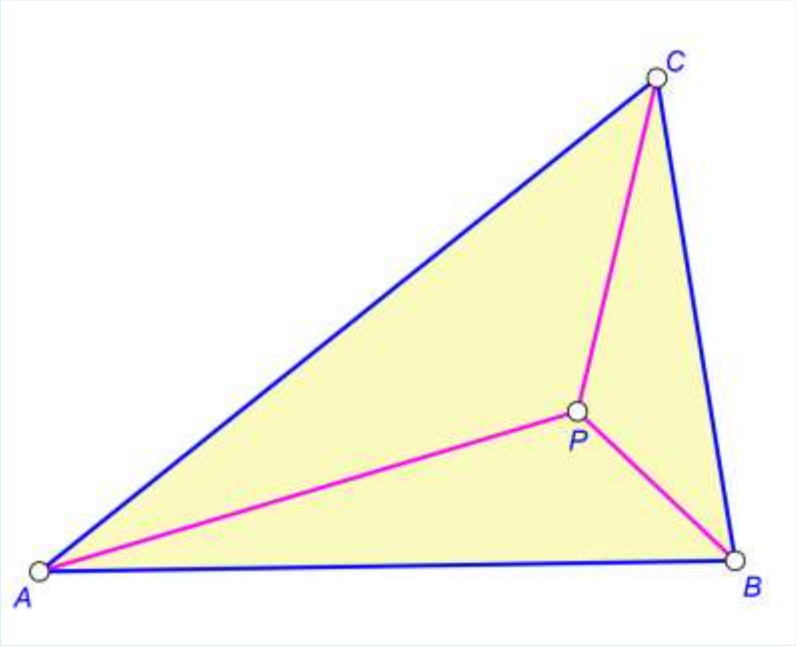
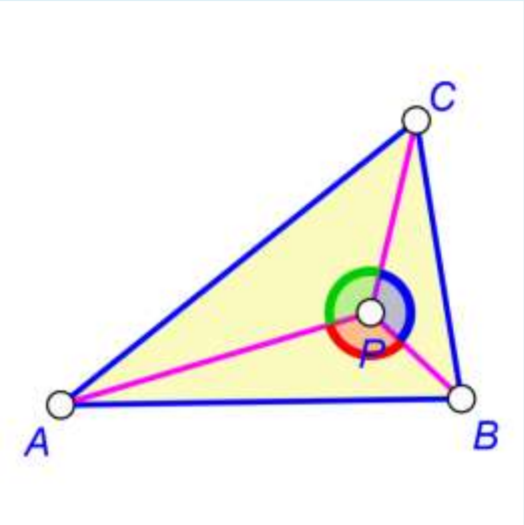
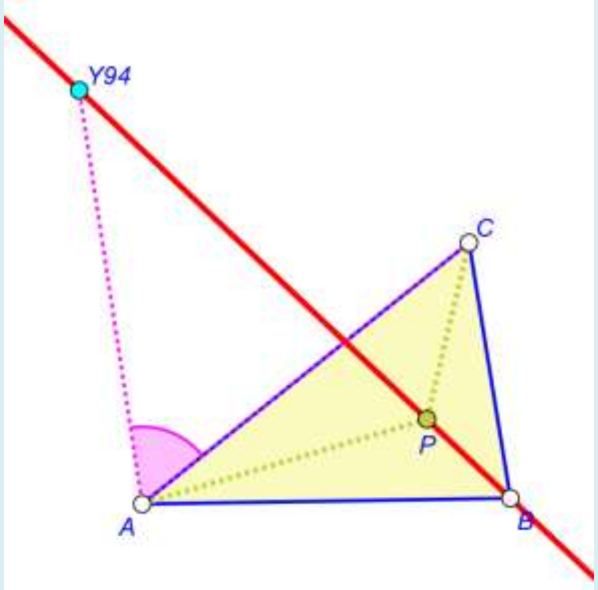
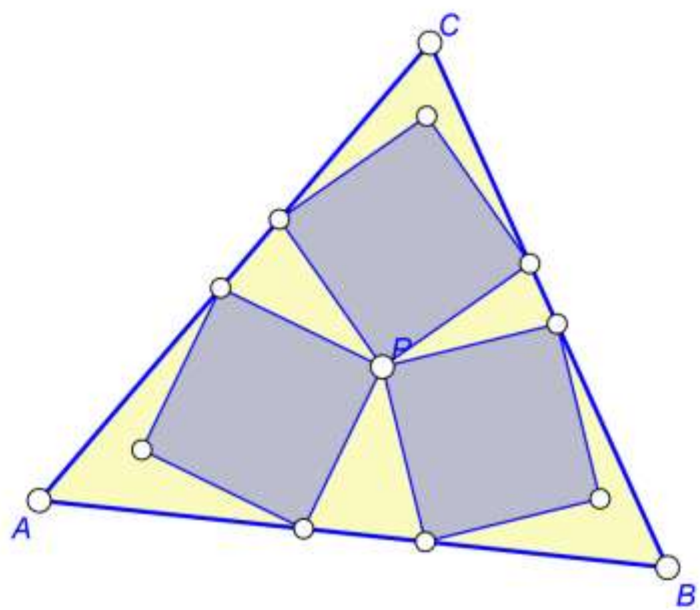
An optimisation problem





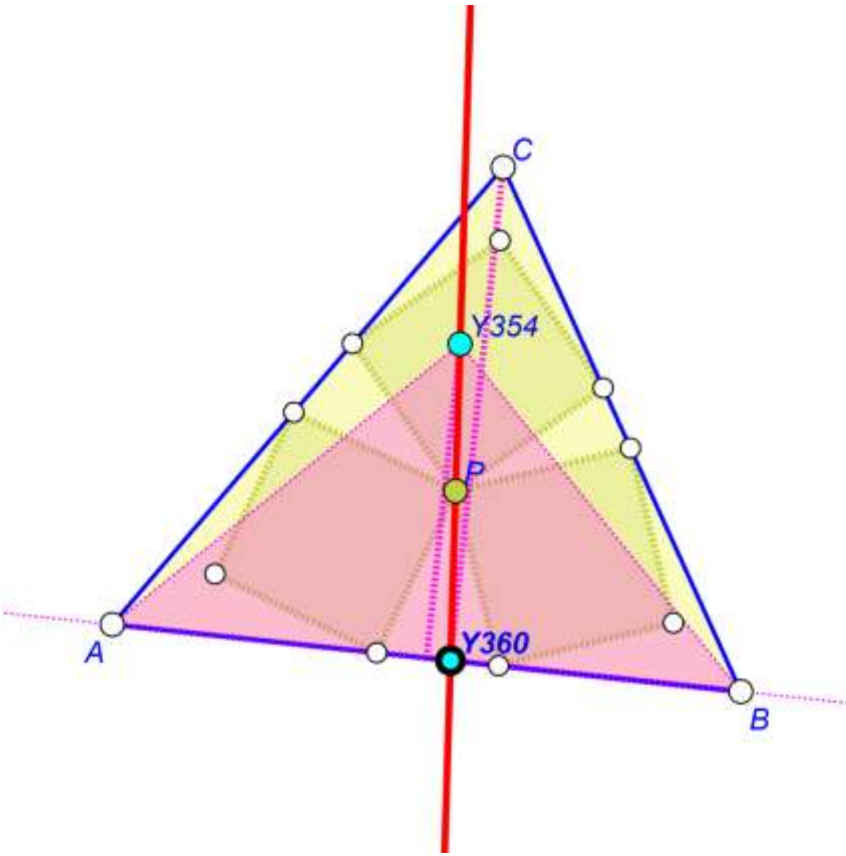


A nice problem



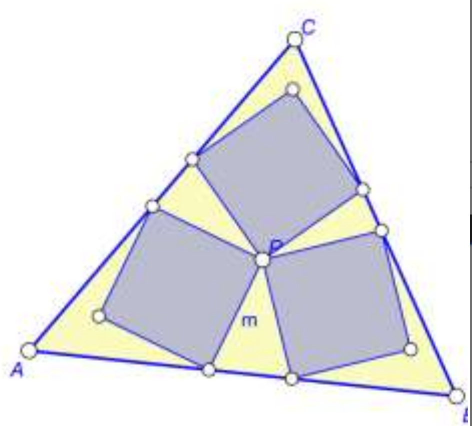
How to inscribe 3 congruent squares into a given triangle ABC as shown in the figure?

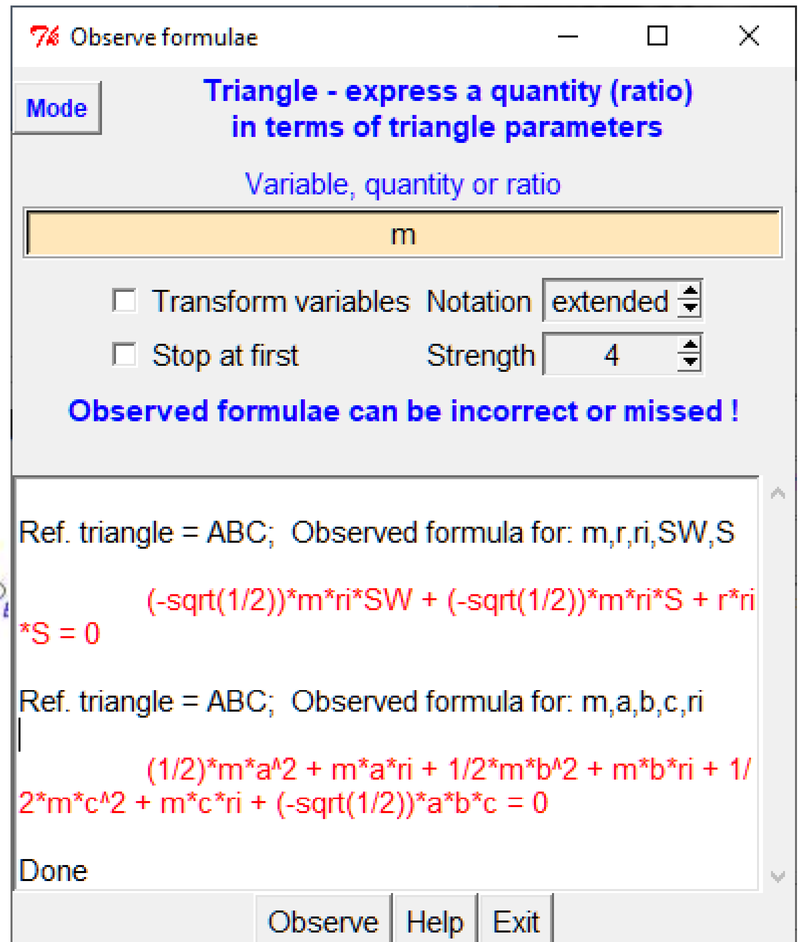
A nice point



- Y354 = Local coordinates x = 1/2, y = 1/2; Object(s): A,B
- Y360 = Projection onto line of point; Object(s): AB,C

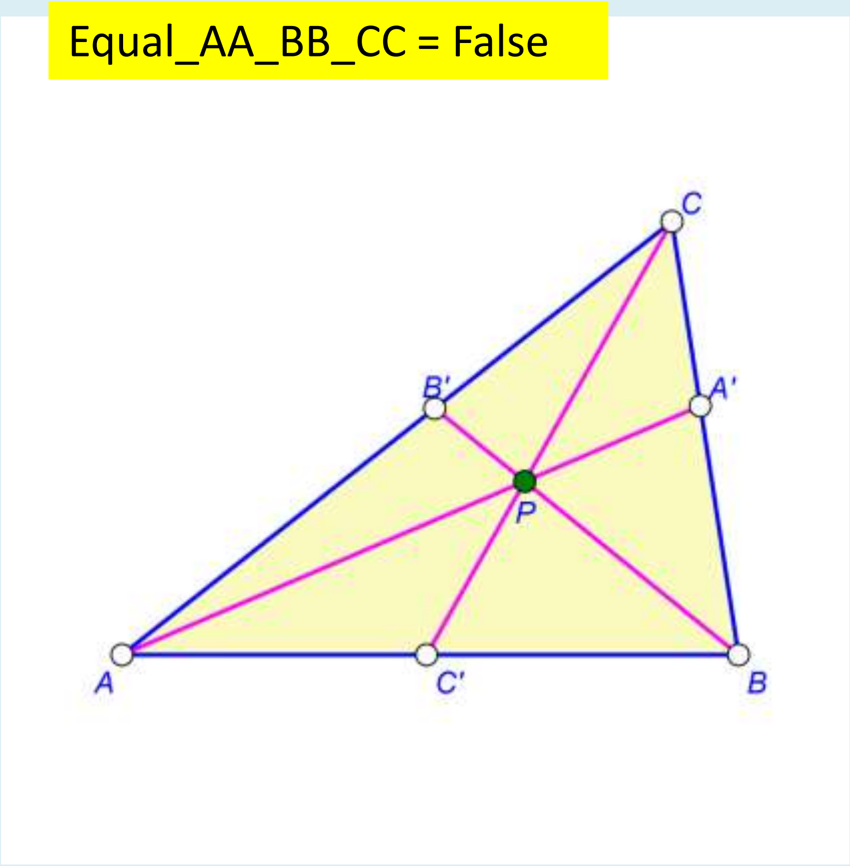
A nice point





Hypothetise the size m of squares in terms of common triangle quantites.

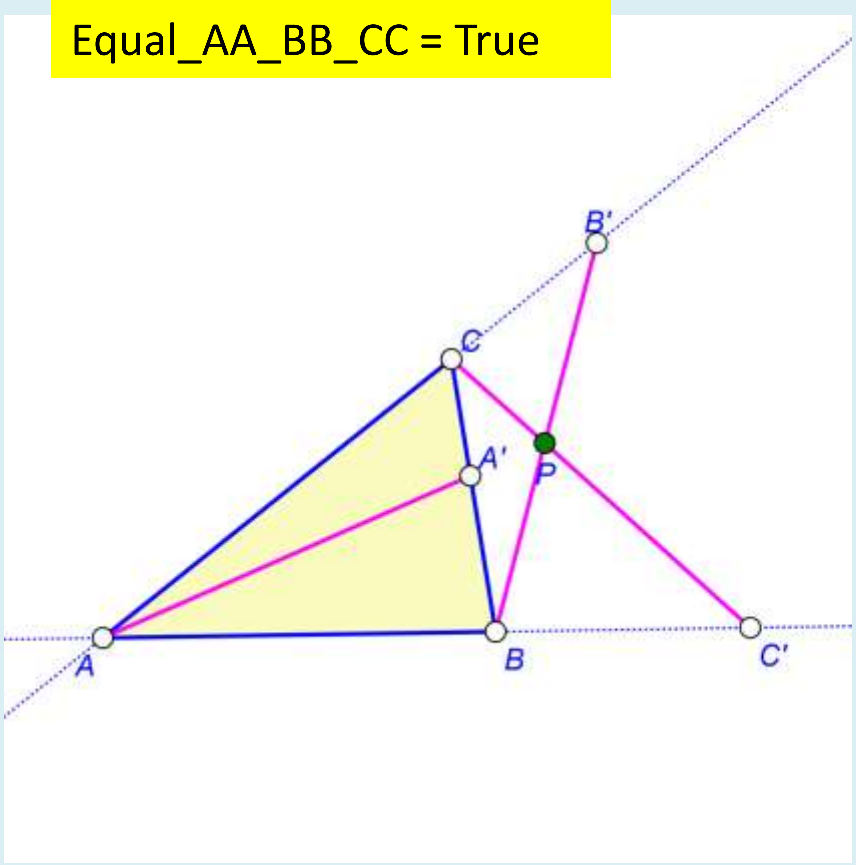
An ‘implicit’ object



- ABC – a triangle
- P – a point
- AA’, BB’, CC’ – Cevian lines of P in ABC.
- **(AA’ ≡ BB’ ≡ CC’)**

Investigate!

An ‘implicit’ problem



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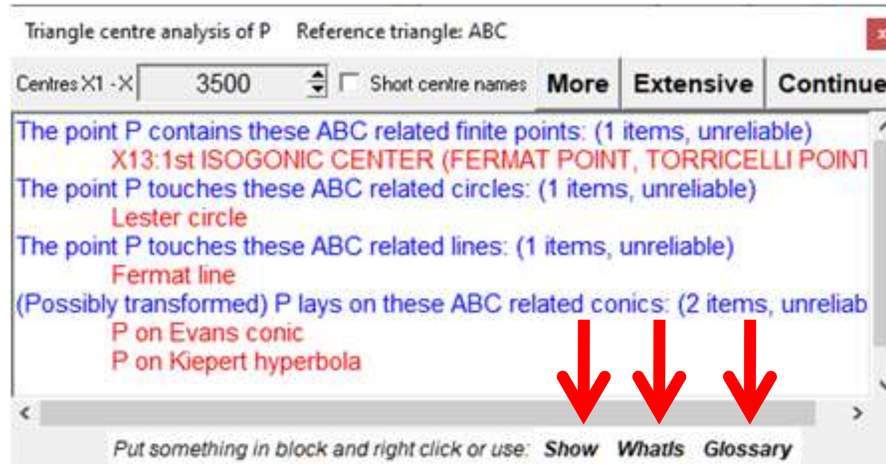
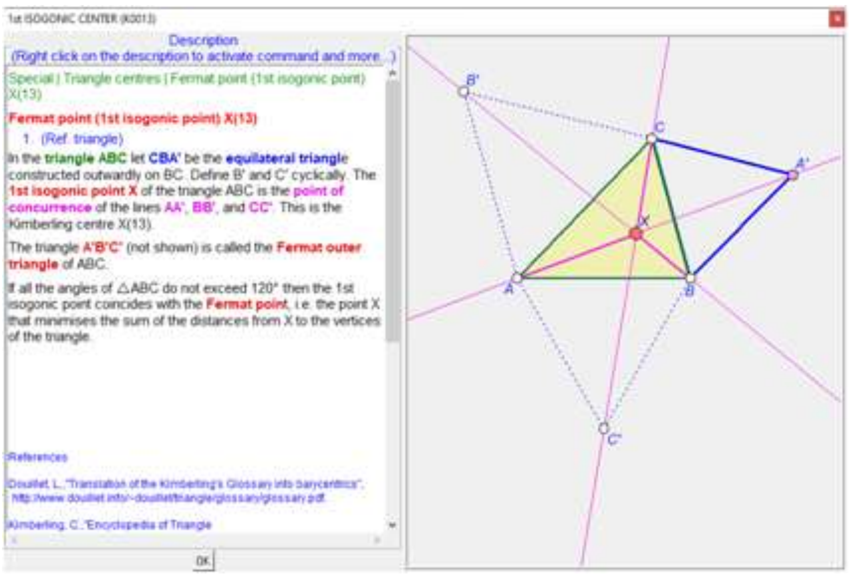
Investigate!

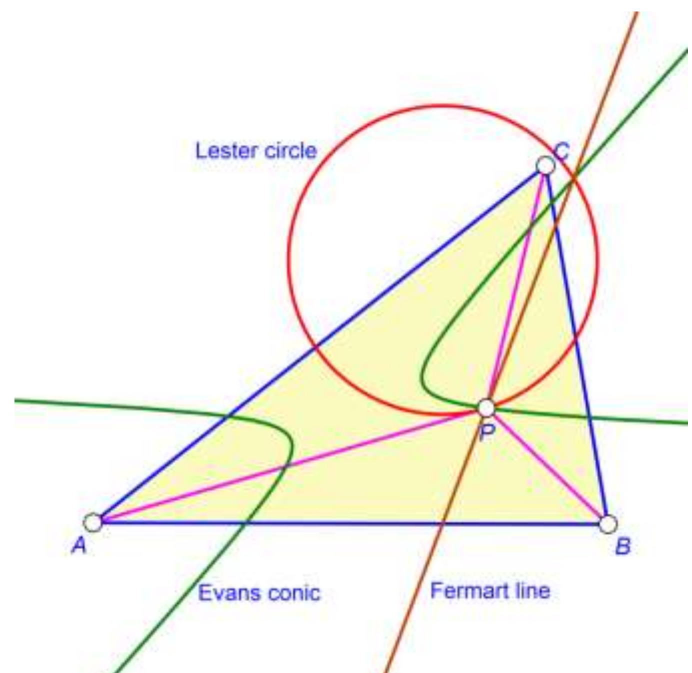
Triangle geometry

- Observe objects wrt. reference triangle
- Drawing triangle objects
- Glossary of triangle objects
- Observing algebraic relations in a triangle

Triangle observation

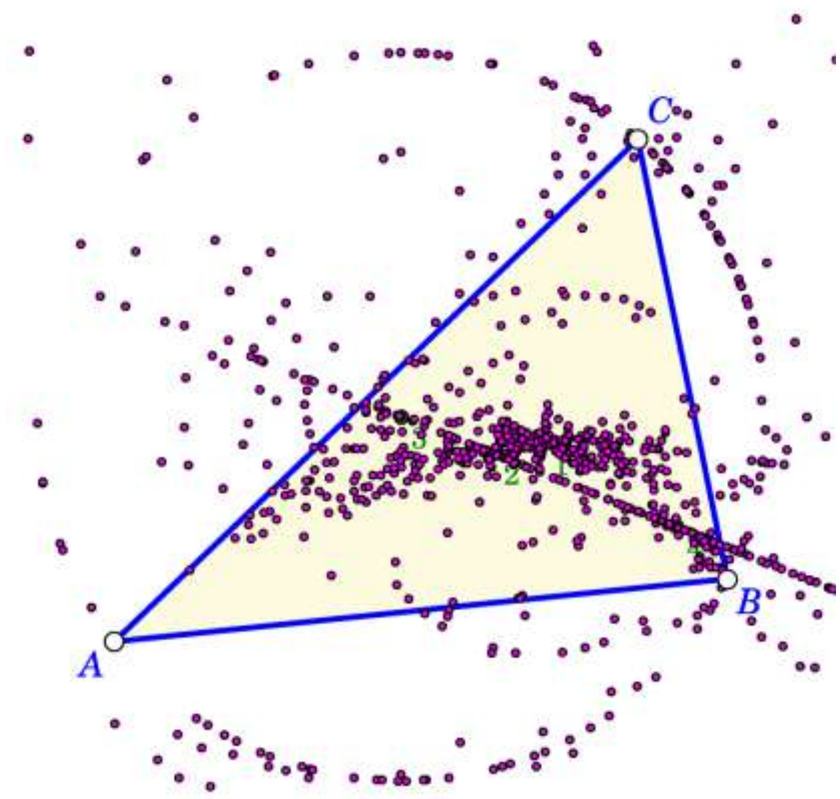






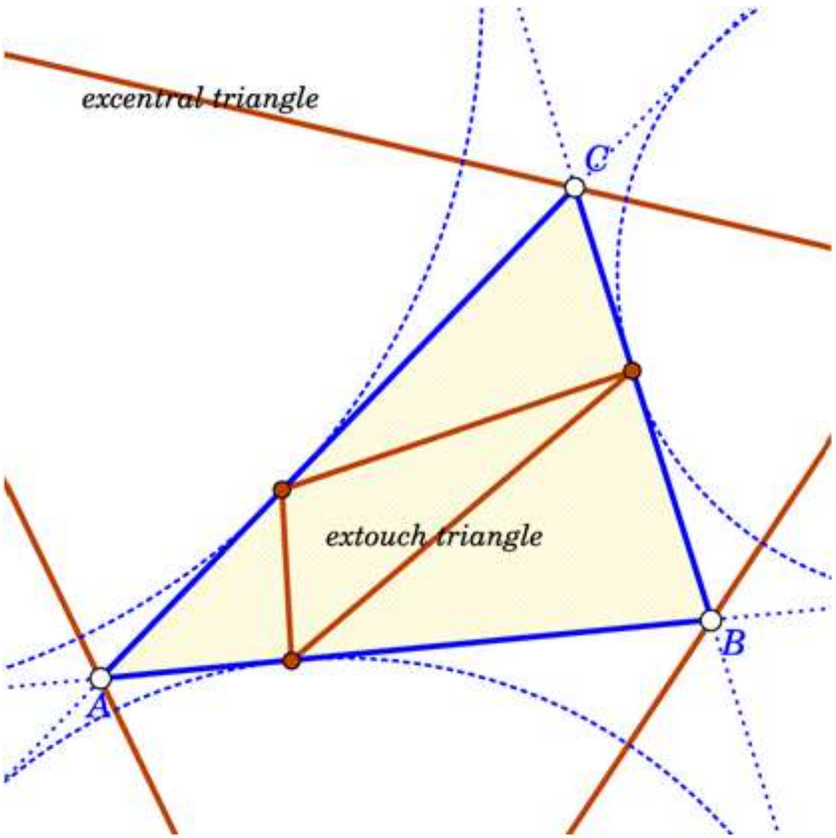
Triangle centres and transformations

- >50.000 centres
- >30 transformations
- ~500.000 transformed centres
- millions of lines connecting the centres

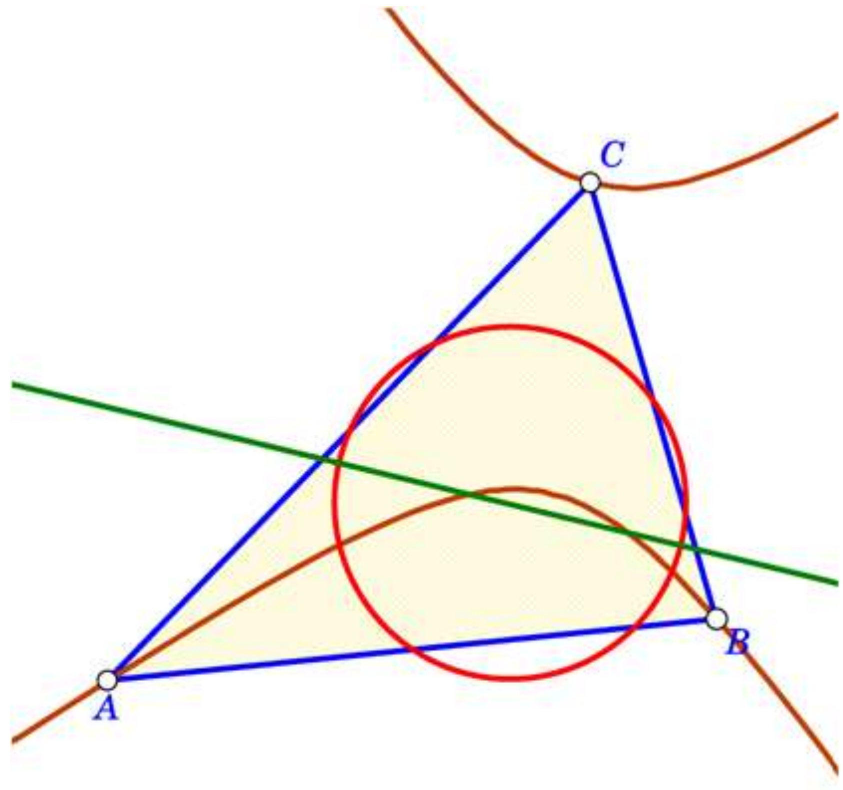


Triangle objects

~ 230 considered triangles →
>2000 lines, >6000 circles

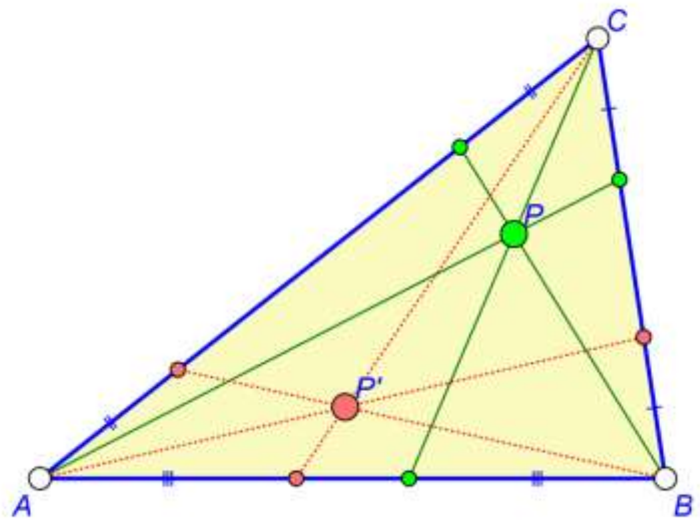


~ 30 considered lines
~100 circles, ~40 conics, ~1300 cubics

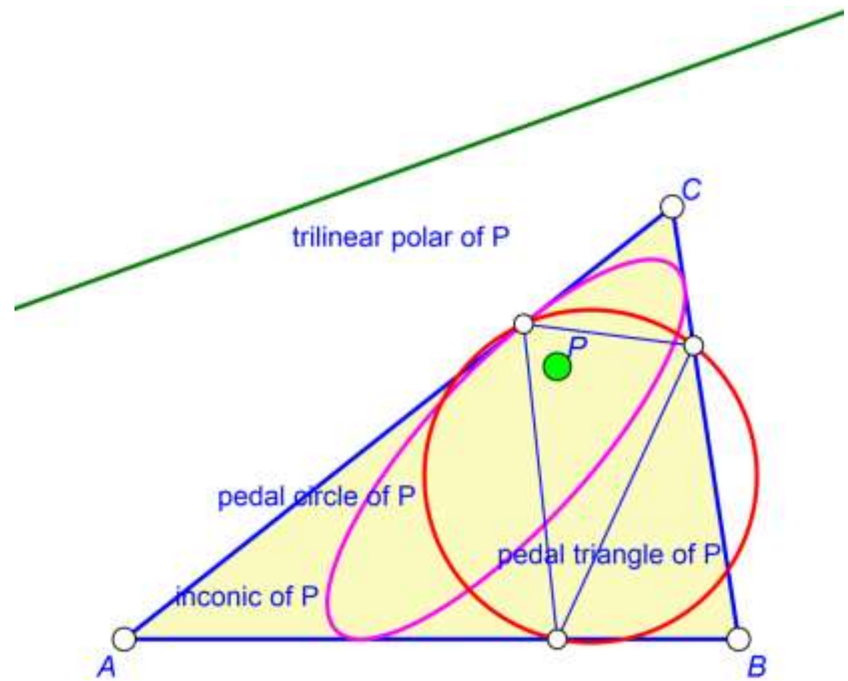


Triangle centres and transformations

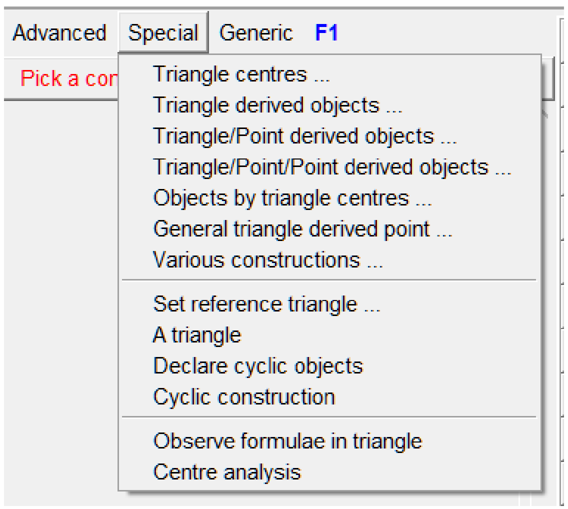
Triangle transformations
(e.g. isotomic conjugation)

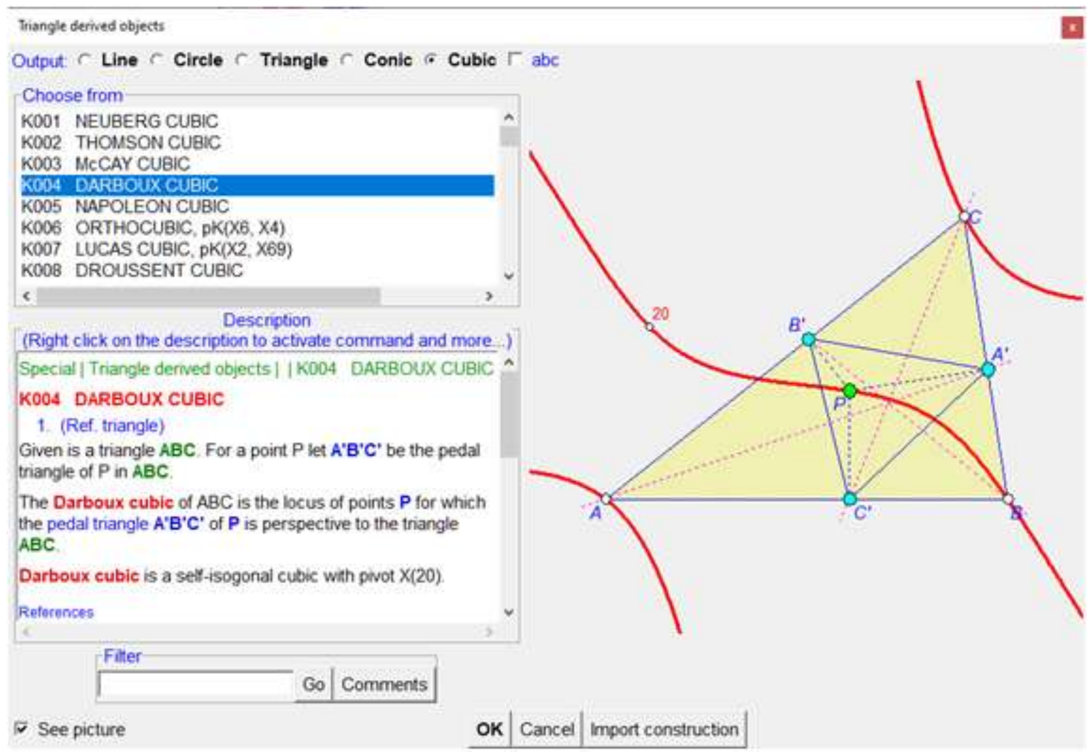


Triangle-Point objects

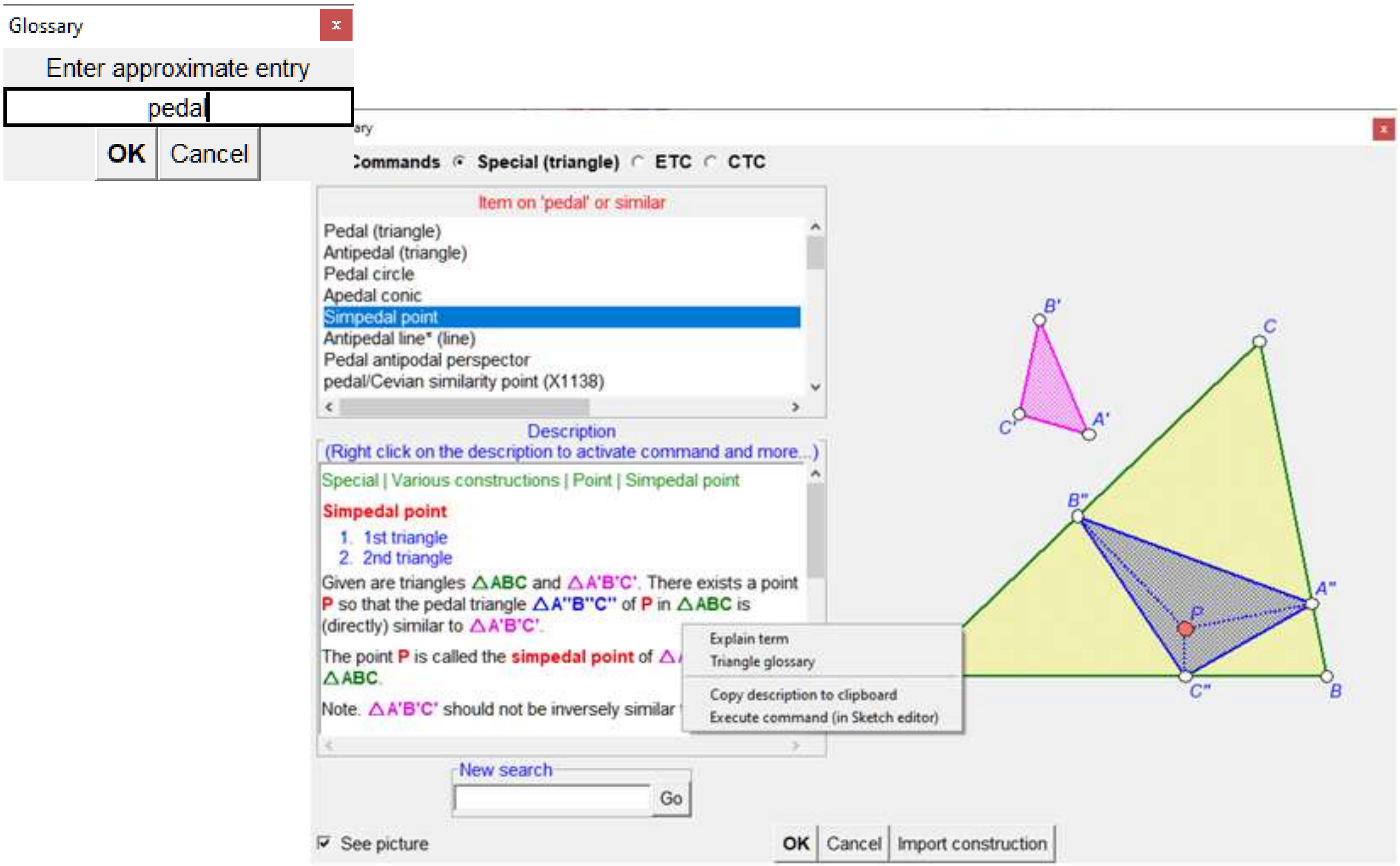


Triangle objects



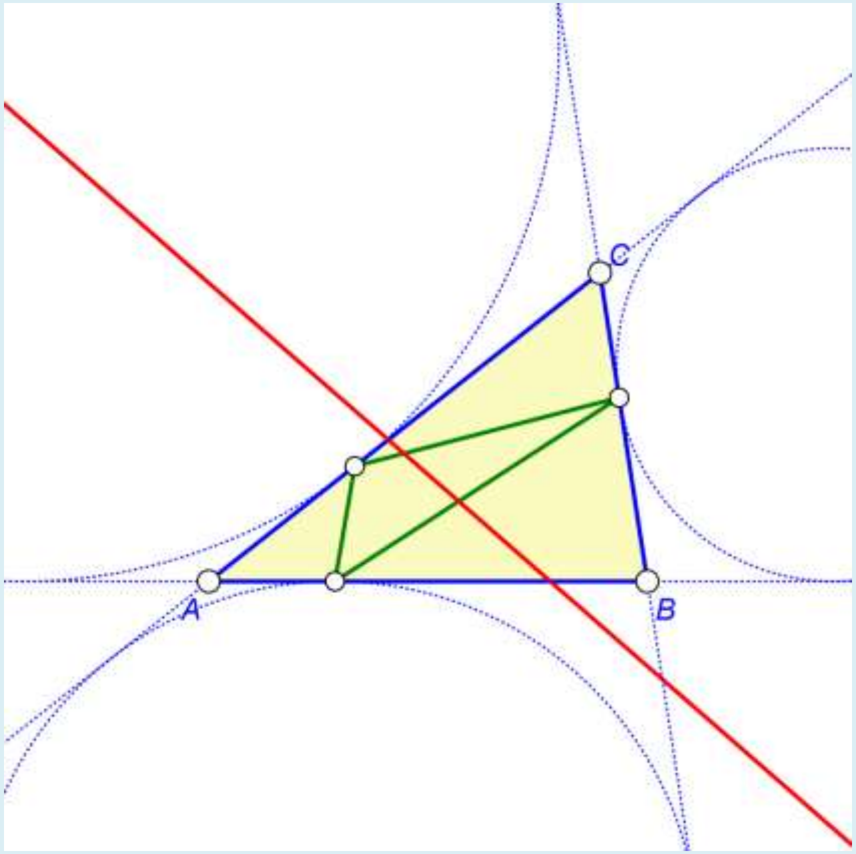


Glossary

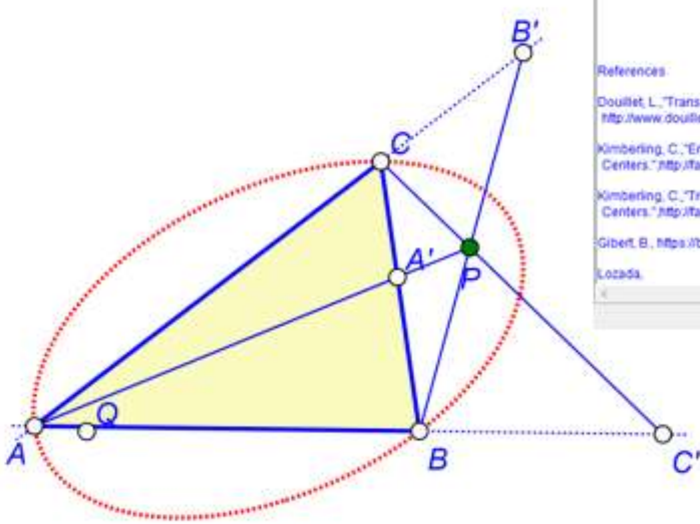

Glossary
Enter approximate entry
pedal
OK
Cancel
Commands
Special (triangle)
ETC
CTC
Item on 'pedal' or similar
Pedal (triangle)
Antipedal (triangle)
Pedal circle
Apedal conic
Simpedal point
Antipedal line* (line)
Pedal antipodal perspector
pedal/Cevian similarity point (X1138)
Description
(Right click on the description to activate command and more...)
Special | Various constructions | Point | Simpedal point
Simpedal point
1. 1st triangle
2. 2nd triangle
Given are triangles △ABC and △A'B'C'. There exists a point P so that the pedal triangle △A"B"C" of P in △ABC is (directly) similar to △A'B'C'.
The point P is called the simpedal point of △
△ABC.
Note. △A'B'C' should not be inversely similar
Explain term
Triangle glossary
Copy description to clipboard
Execute command (in Sketch editor)
New search
Go
See picture
OK
Cancel
Import construction
B'
C'
A'
C
B"
A"
P
C"
B

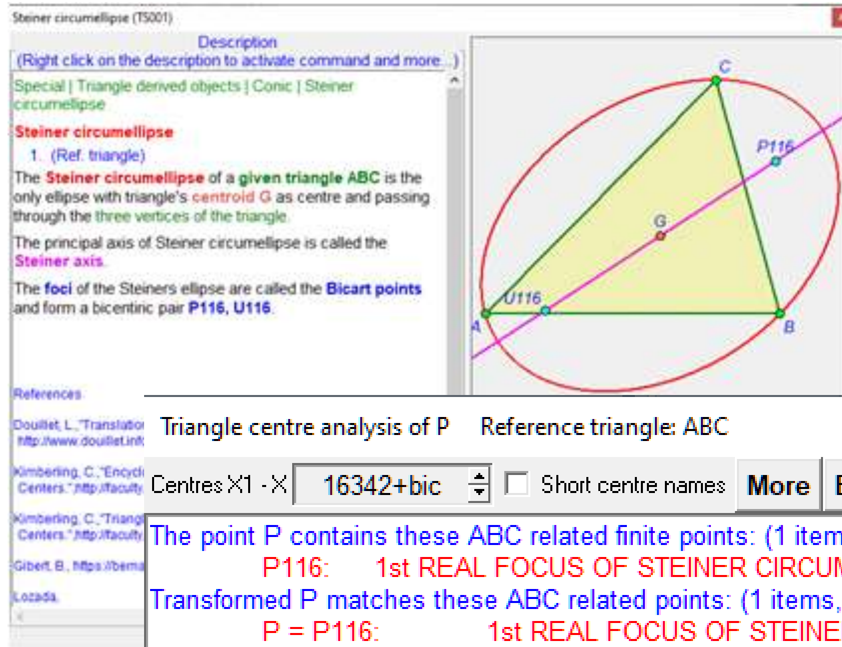
Triangle objects

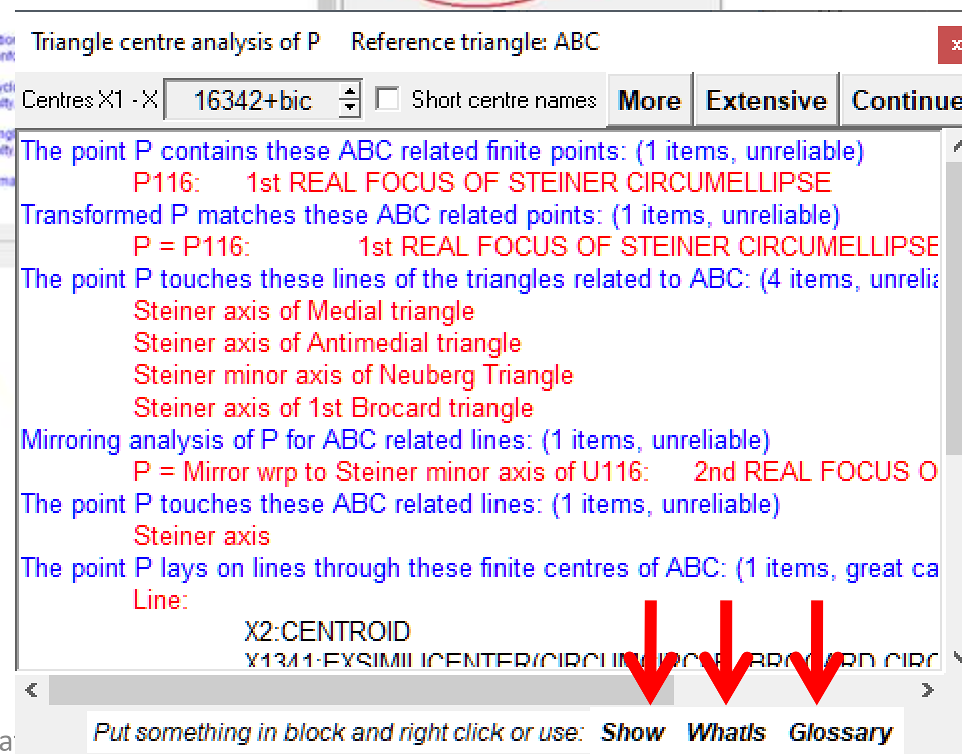
- Given is a triangle ABC.
- Draw the Euler line of the extouch triangle of ABC.



Congruent Cevians

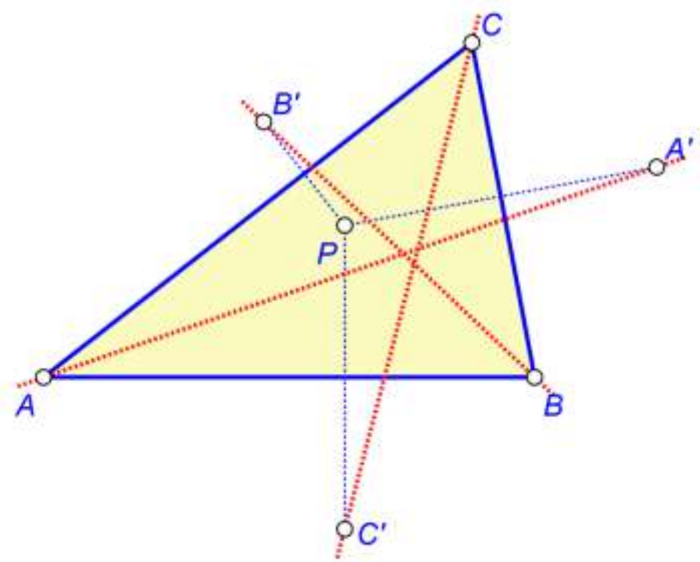






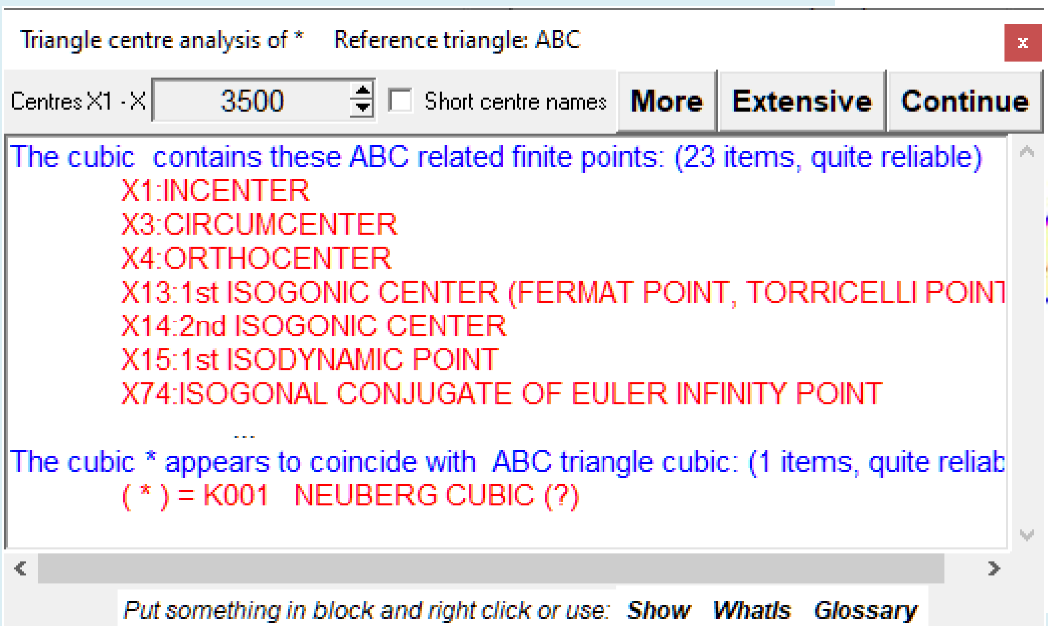
Example of a triangle locus

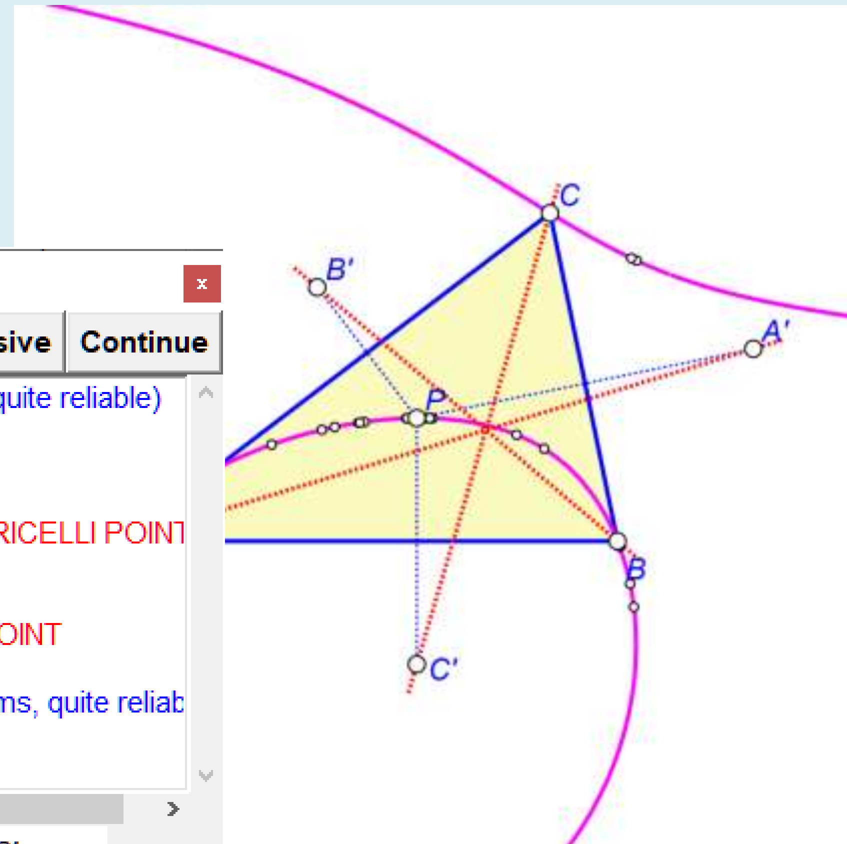
- A',B',C' are the mirror images of a point P in the sides of triangle ABC.
- For what points P are the lines AA', BB', CC' concurrent?



Example of a triangle locus

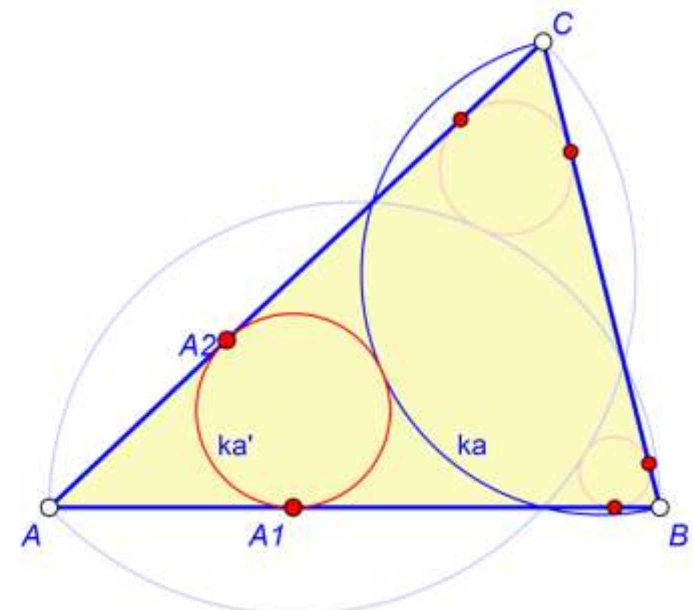
- A’,B’,C’ are the mirror images of a point P in the sides of triangle
-





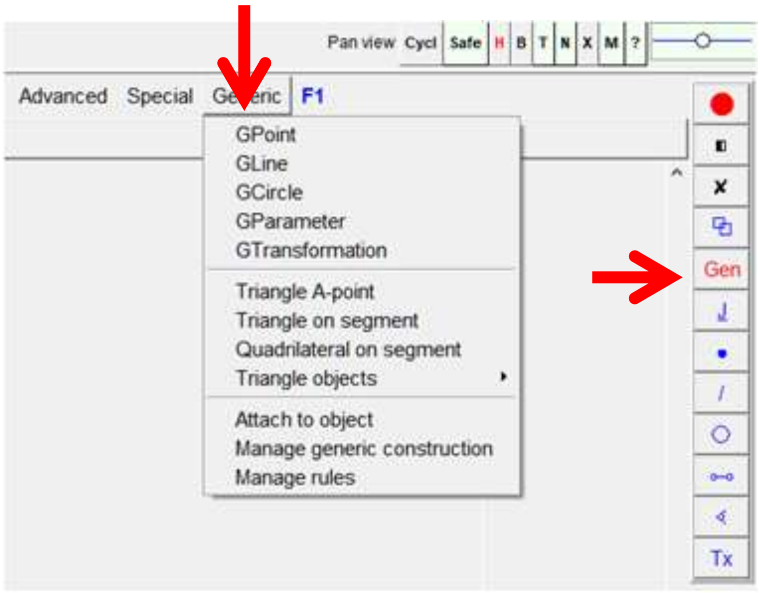
Cyclic constructions

- ABC – an acute triangle
- *ka* – the inwards semicircle on BC
- *ka'* – the smallest of circles touching AB, AC, and (externally) *ka*
- *kb', kc'* – defined cyclically
- Investigate the points of contact of *ka', kb', kc'* with the sidelines of ABC.



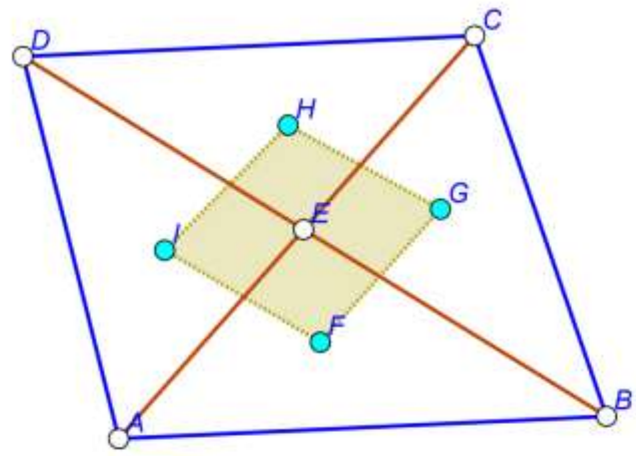
Generic constructions

- Generic constructions are constructionally isomorphic families of dynamic constructions.
- Generic constructions appear and behave like ordinary constructions, in which some construction steps consist of rules (i.e. groups of isomorphic operations).



All resulting constructions can be visualised, analysed, checked for properties, etc. at the same time.

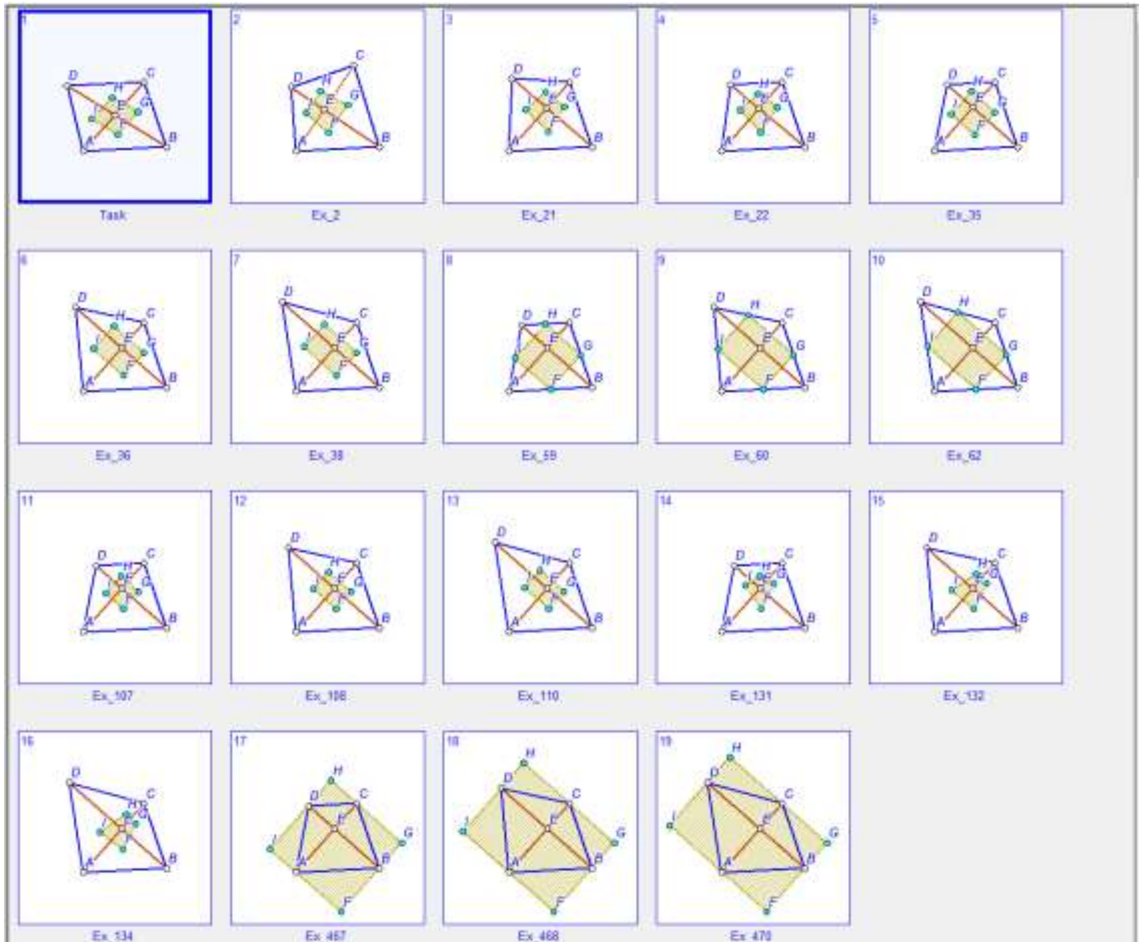
Generic constructions – constructionally isomorphic configurations



1. ABCD – a **trapezium**
2. E – AC ∩ BD
3. F, G, H, I **incentres** of the 4 triangles (ABE,BCE,CDE,DAE)

1. **Quadrilateral →** **Random, bicentric, cyclic, equidiagonal, ….**)

3. **incentres** → **incentre, centroid, circumcentre, orthocentre, …**

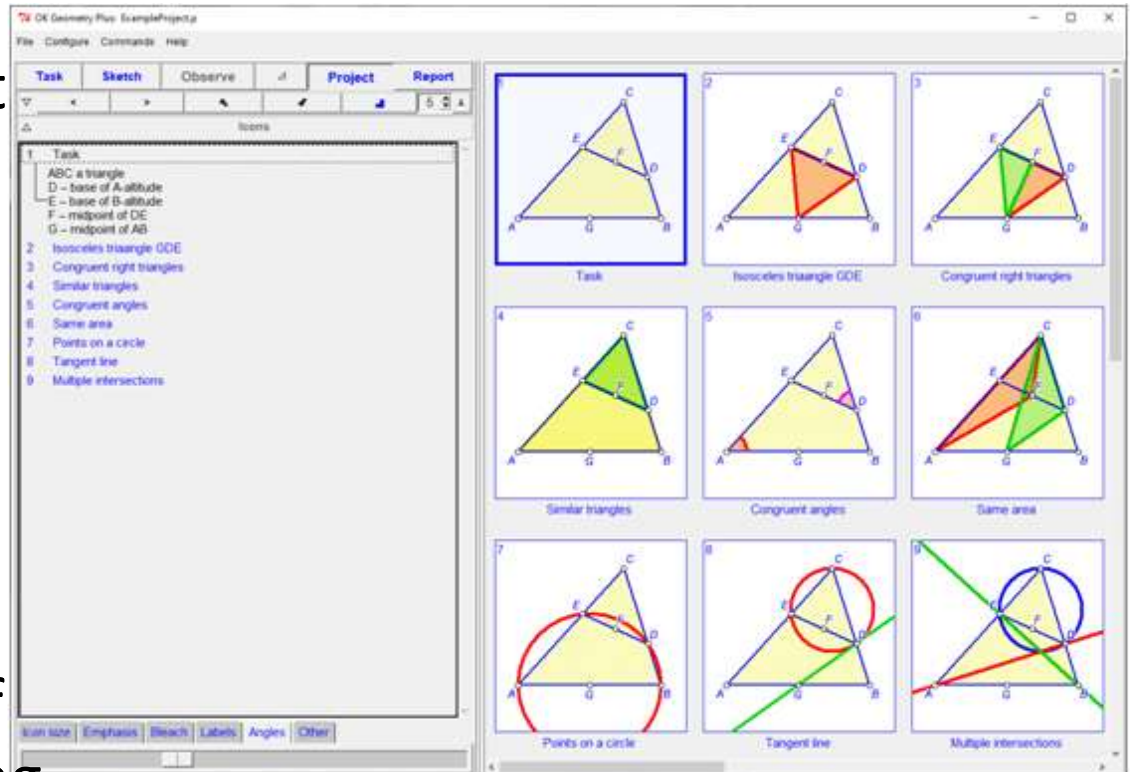


Shaded 4-laterals are cyclic in 43 cases out of 485 checked, e.g.:

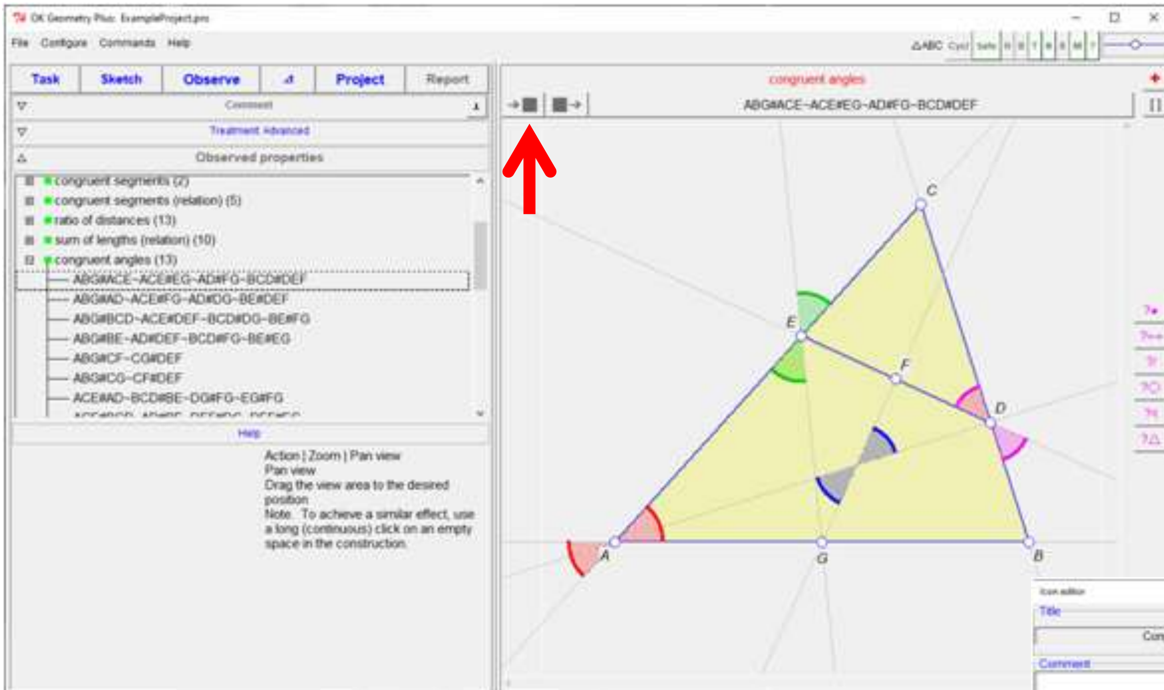
- incentres for bicentric quadrilaterals,
- 9-point centres for Pytagorean quadrilateral…

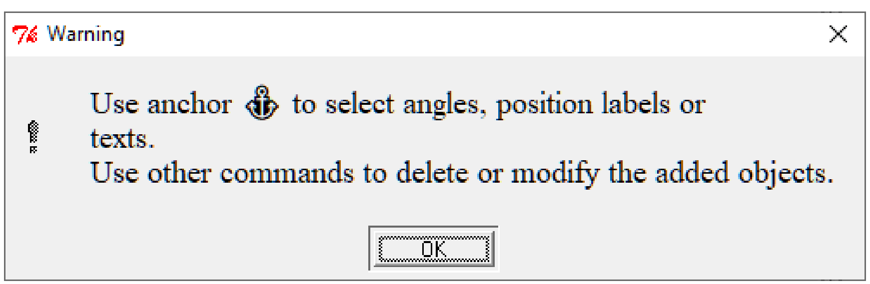
Projects

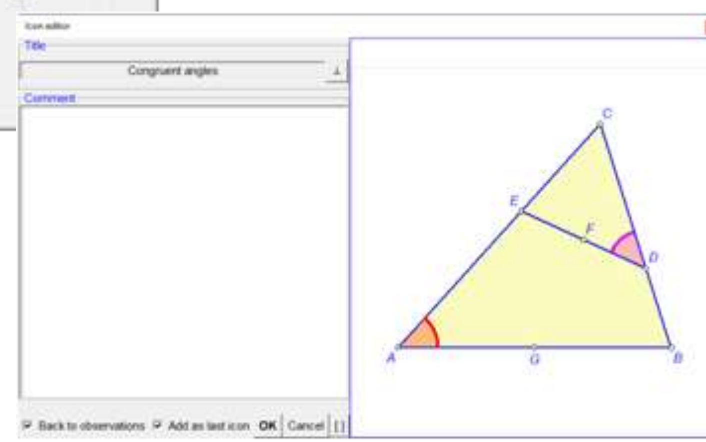
- You can collect constructions, part of constructions, results, etc. into a project.
- A project may contain related constructions, observed properties, a proof of a claim, a proving task, etc.



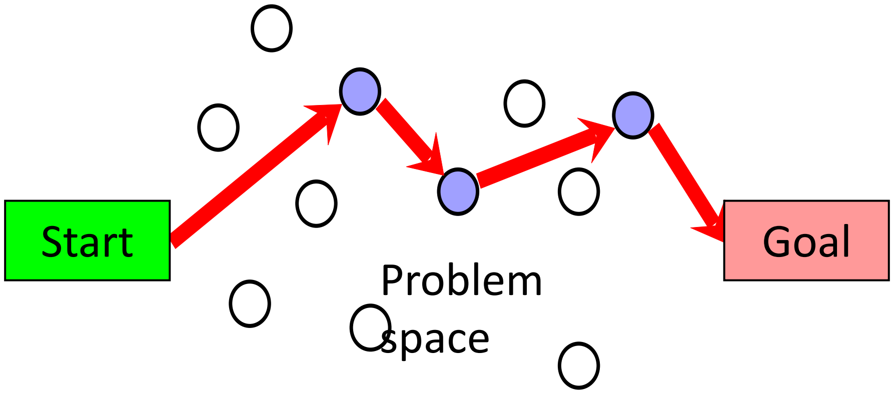
Saving properties



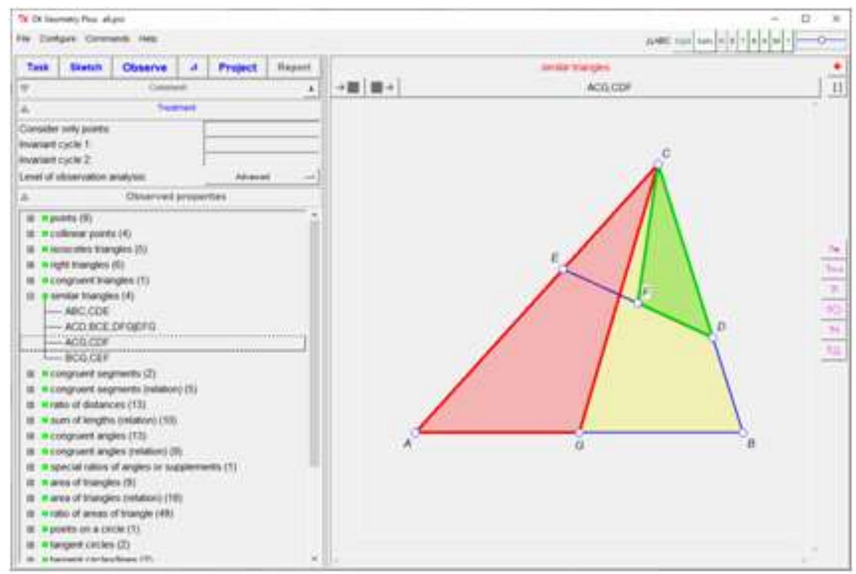




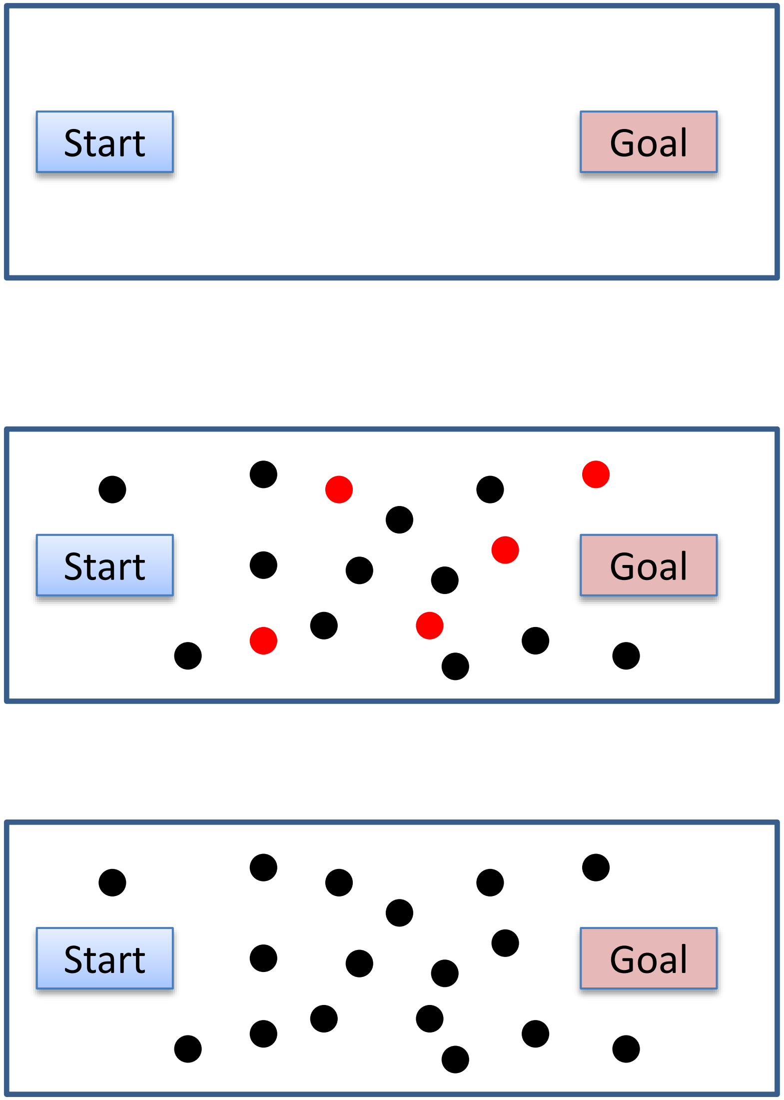
Proving tasks



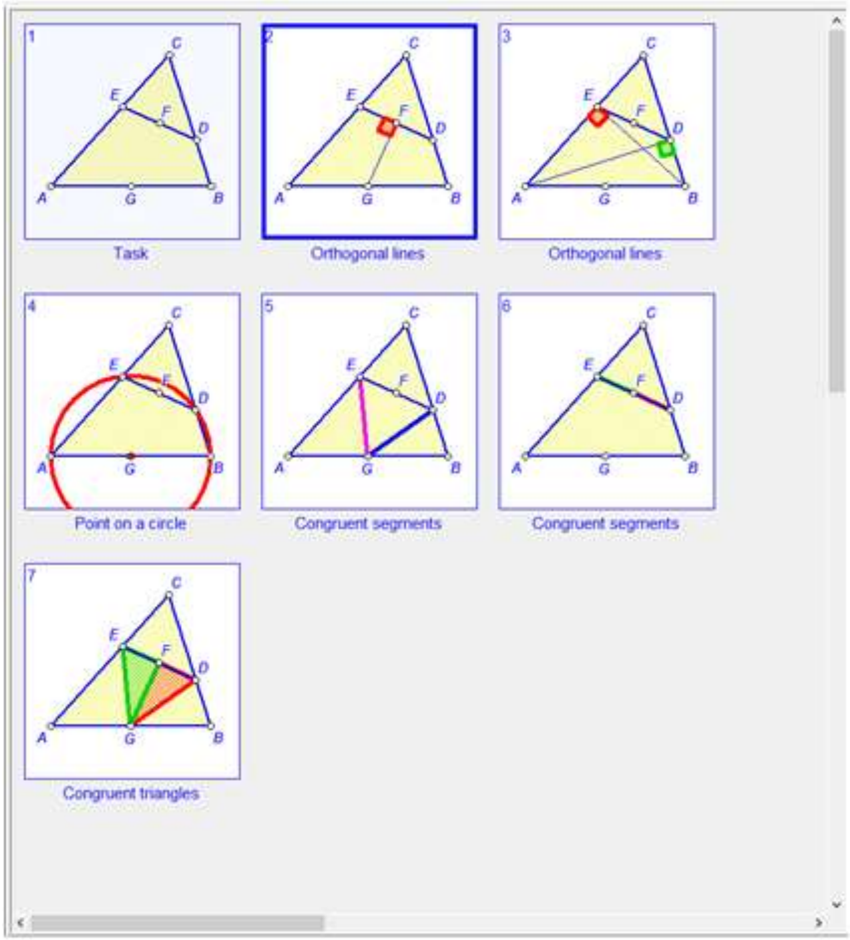
- Observe properties
- Select relevant properties
- Organise the properties
- Provide deductive argumentation

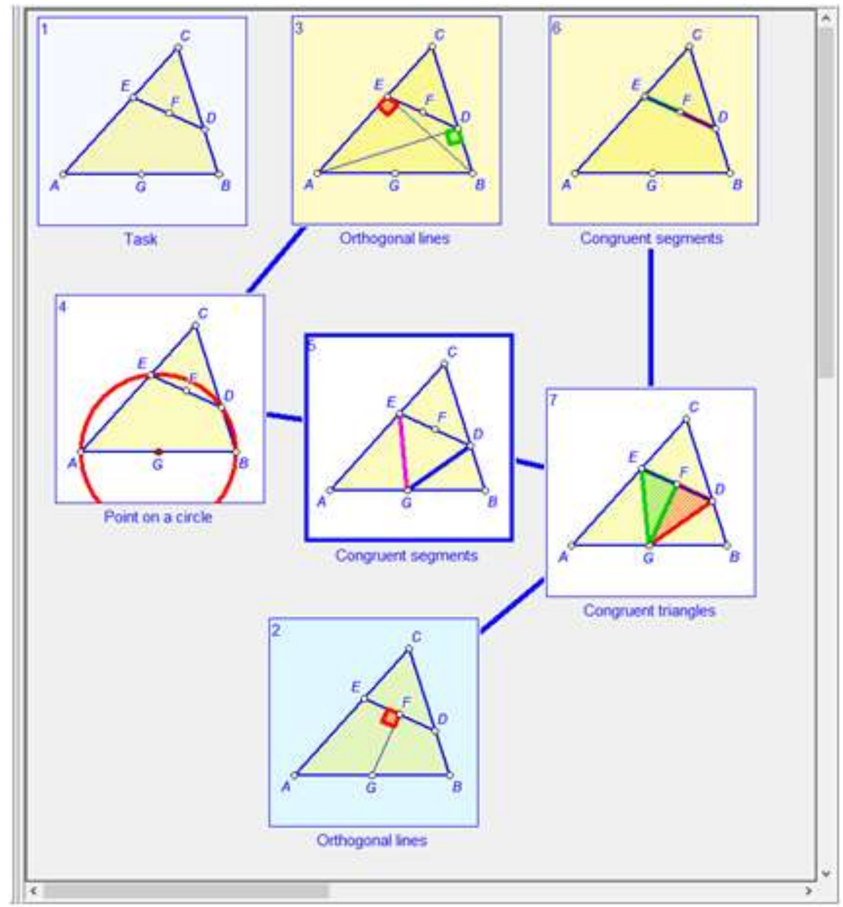


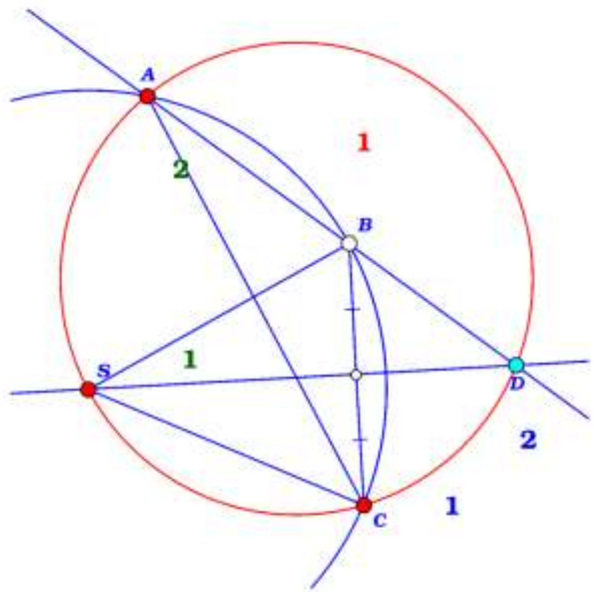
Does a given problem space help?



Proving


Task
Orthogonal lines
Orthogonal lines
Point on a circle
Congruent segments
Congruent segments
Congruent triangles


Task
Orthogonal lines
Congruent segments
Point on a circle
Congruent segments
Congruent triangles
Orthogonal lines



Task

1 Task

Given is a circle with centre S and and three points, A, B, C on its circumference. Let D be the intersection of the line AB and the bisector of the chord BC.

Prove that S, C, D, and A are cocyclic.

Comment:

2 Proof

Definition Let E be the midpoint of BC.

Claim 1 ∠CSB = 2·∠CSD

Argument 1 First, note that S lays on the bisector of segment BC (since |SB|=|SC|). Let E be the midpoint of BC. The triangles AEB and SEC are congruent by sss. Thus

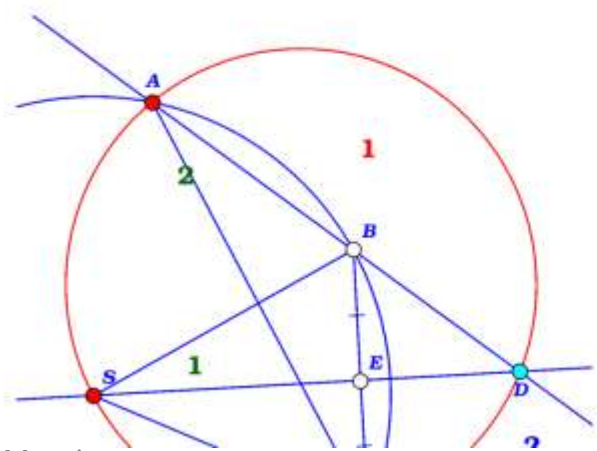
∠CSE = ∠ESB

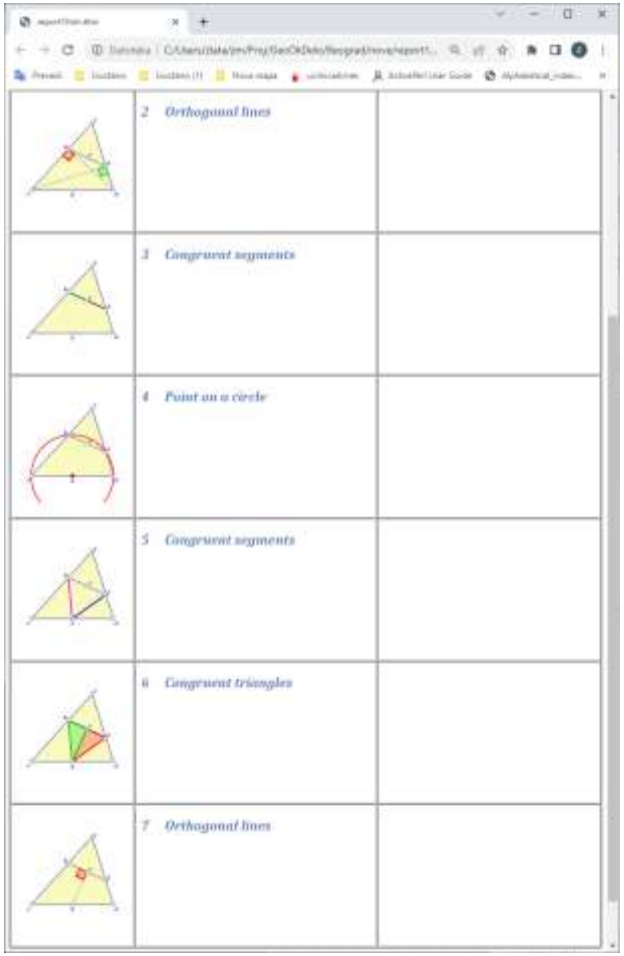
and consequently

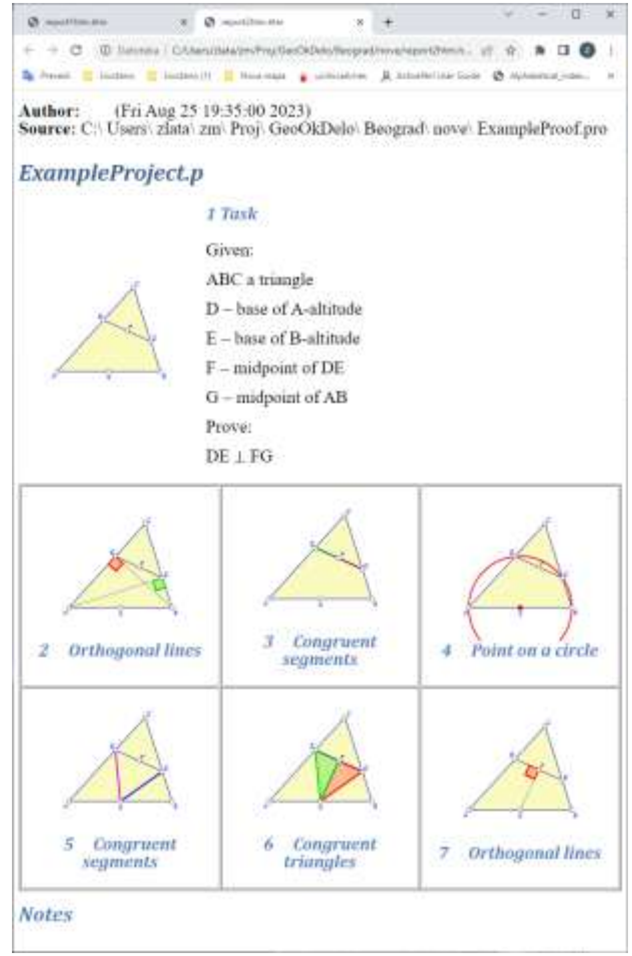
∠CSB = 2·∠CSD.

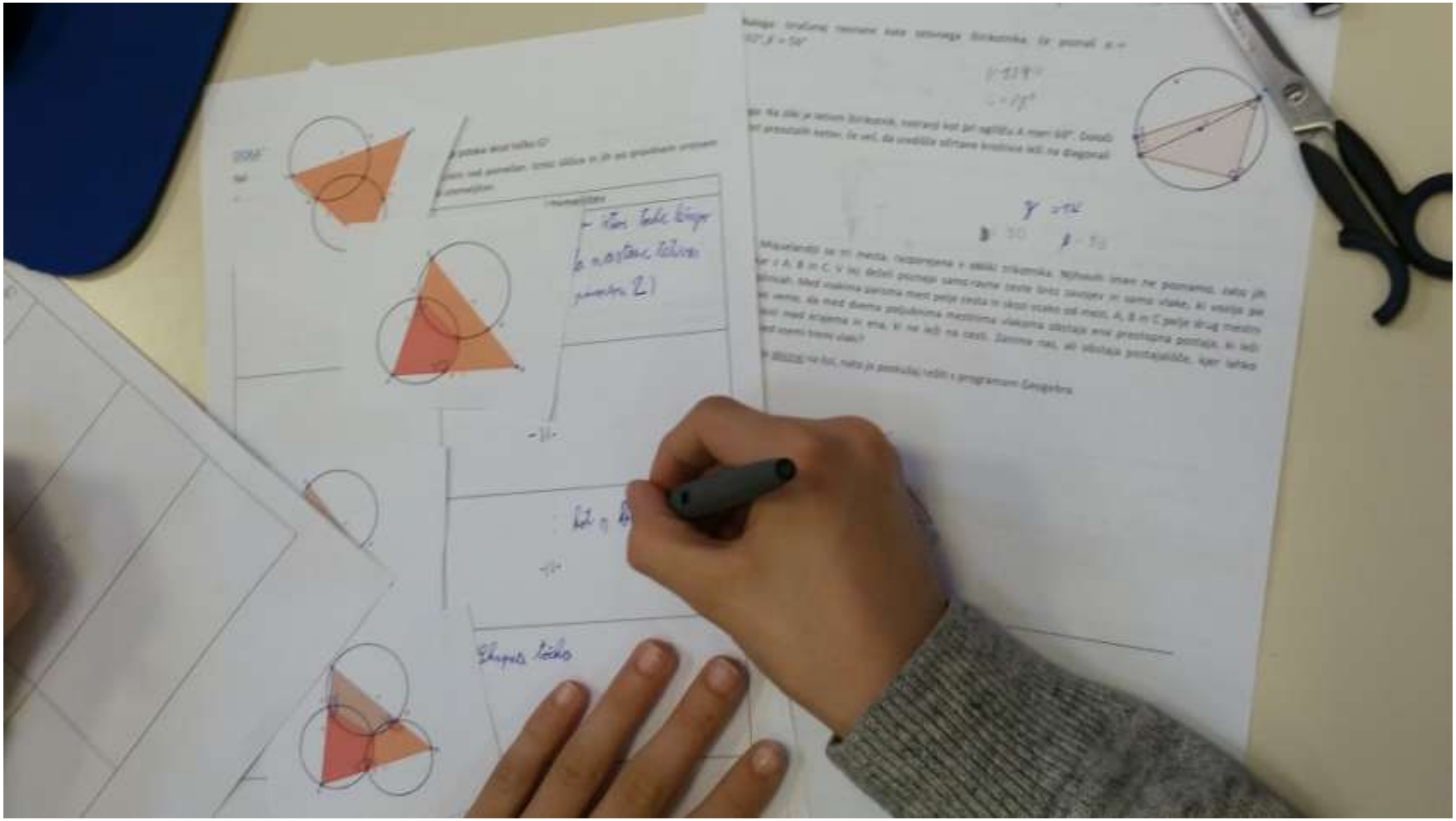
Claim 2 ∠CAB = ∠CSD

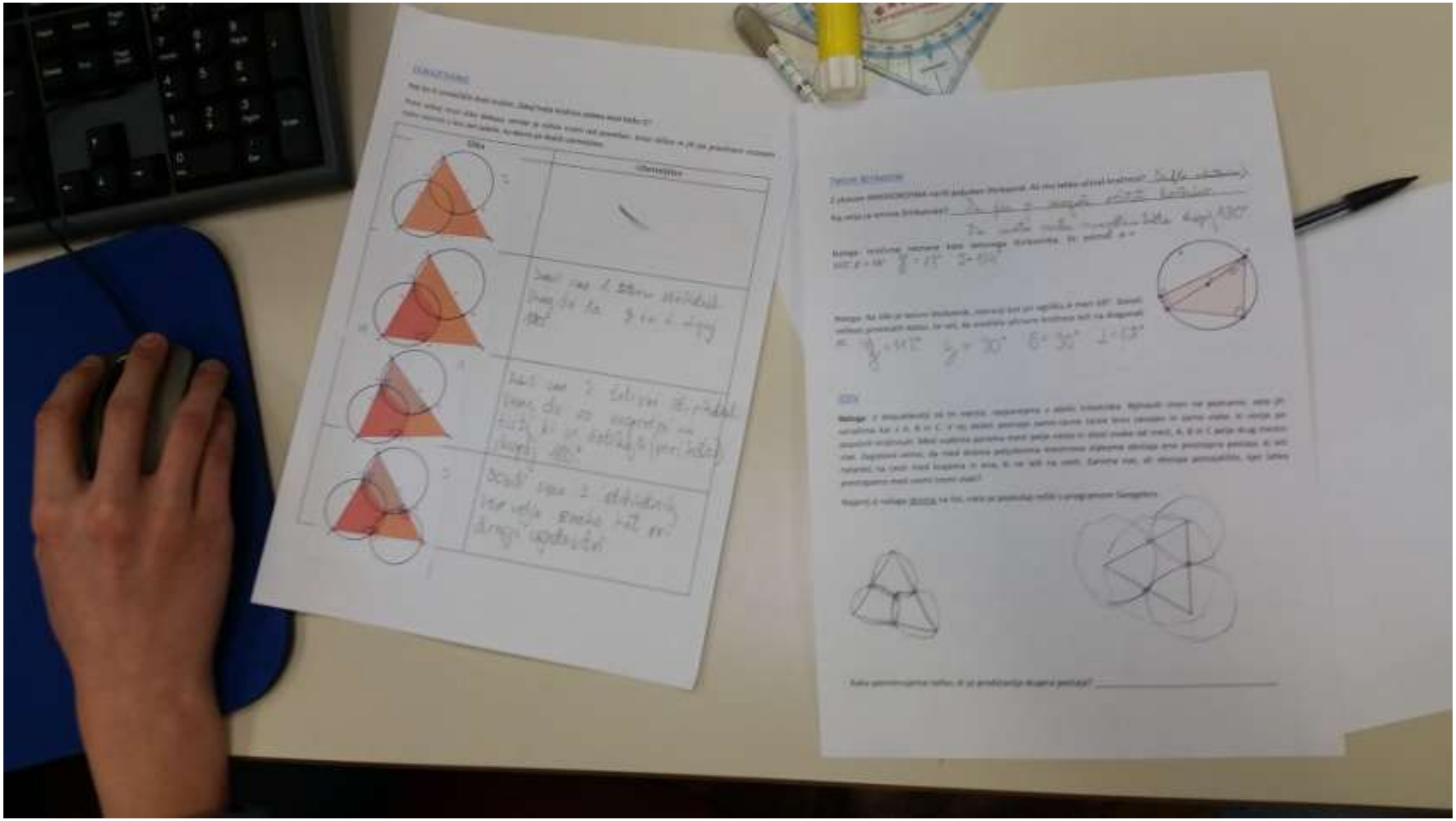
Argument 2 The arc BC of the circle k(S,A) spans an inscribed angle ∠CAB and the central angle ∠CSB. By a known theorem



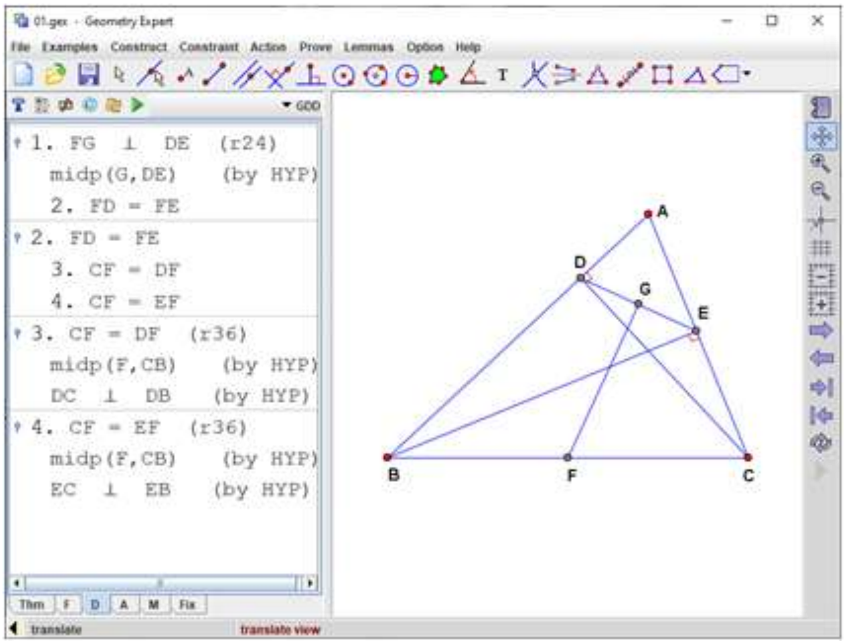

2 Orthogonal lines
3 Congruent segments
4 Point on a circle
5 Congruent segments
6 Congruent triangles
7 Orthogonal lines

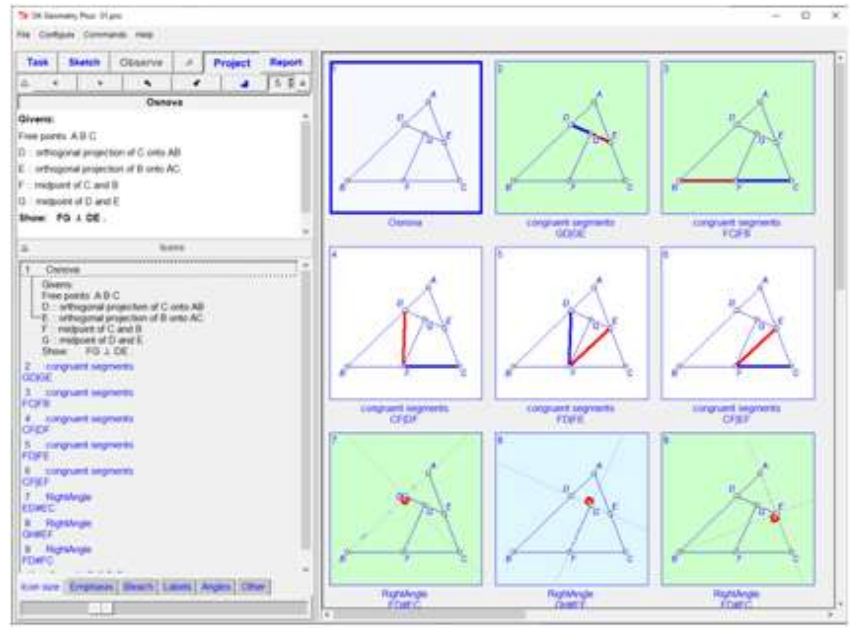

Author: (Fri Aug 25 19:35:00 2023)
Source: C:\ Users\ zlata\ zm\ Proj\ GeoOkDelo\ Beograd\ nove\ ExampleProof.pro
ExampleProject.p
1 Task
Given:
ABC a triangle
D – base of A-altitude
E – base of B-altitude
F – midpoint of DE
G – midpoint of AB
Prove:
DE ⊥ FG
2 Orthogonal lines
3 Congruent segments
4 Point on a circle
5 Congruent segments
6 Congruent triangles
7 Orthogonal lines
Notes





Importing proofs
JGEX → OK Geometry

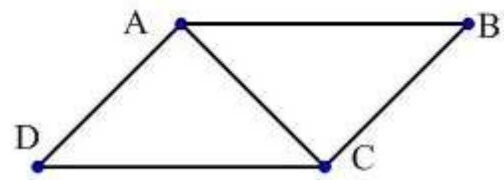




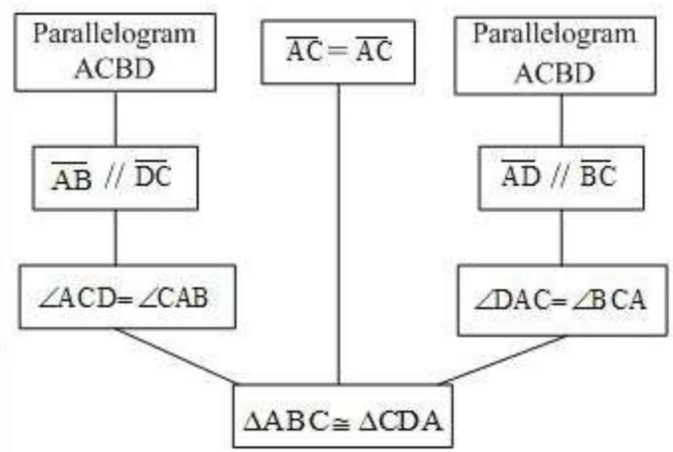
Multiple representations – Mr Geo
(Wong, Yin, Yang, Cheng, 2011)

Given: Parallelogram ABCD with diagonal $\overline{AC}$

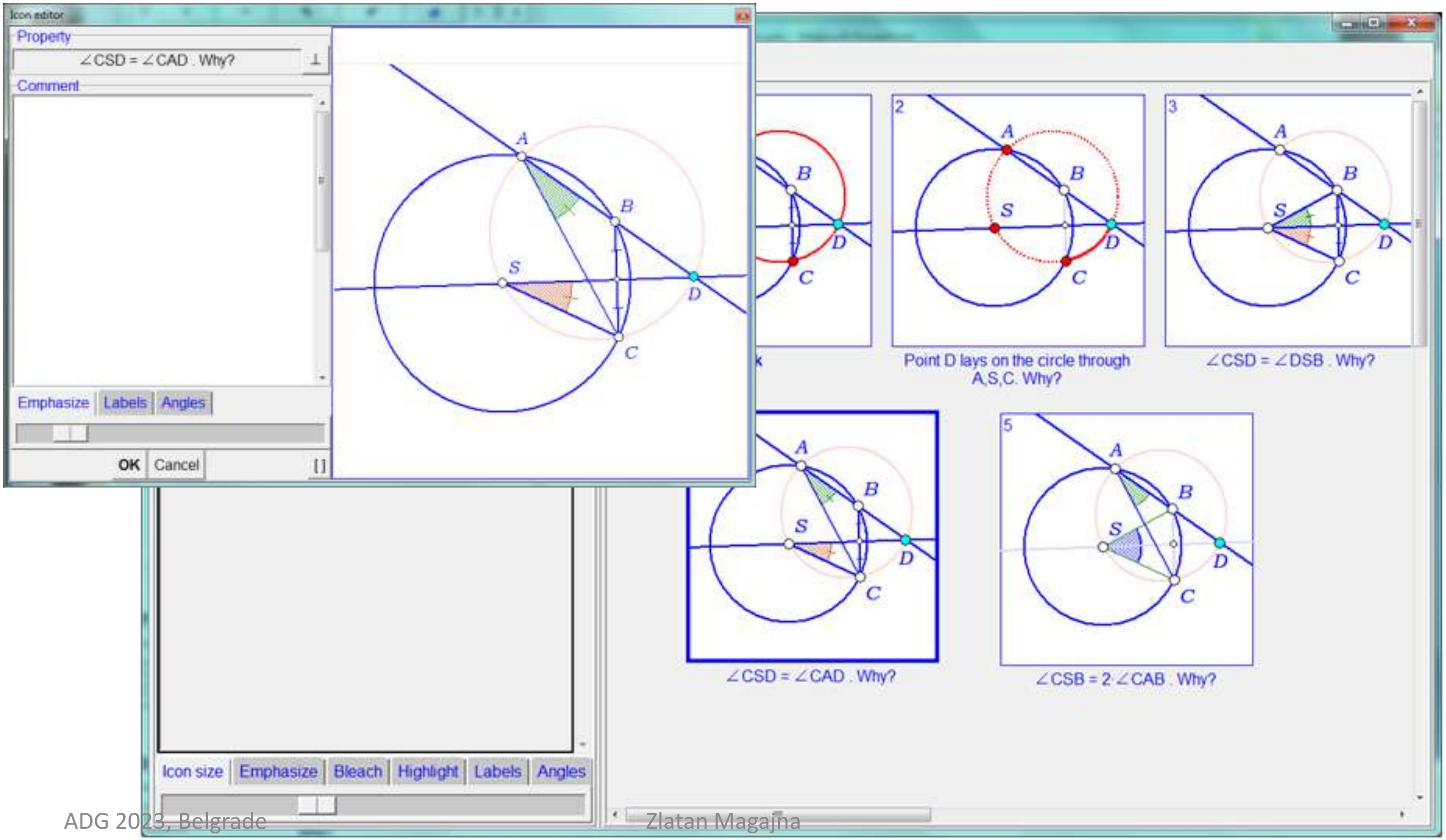
Prove: $\Delta ABC \cong \Delta CDA$



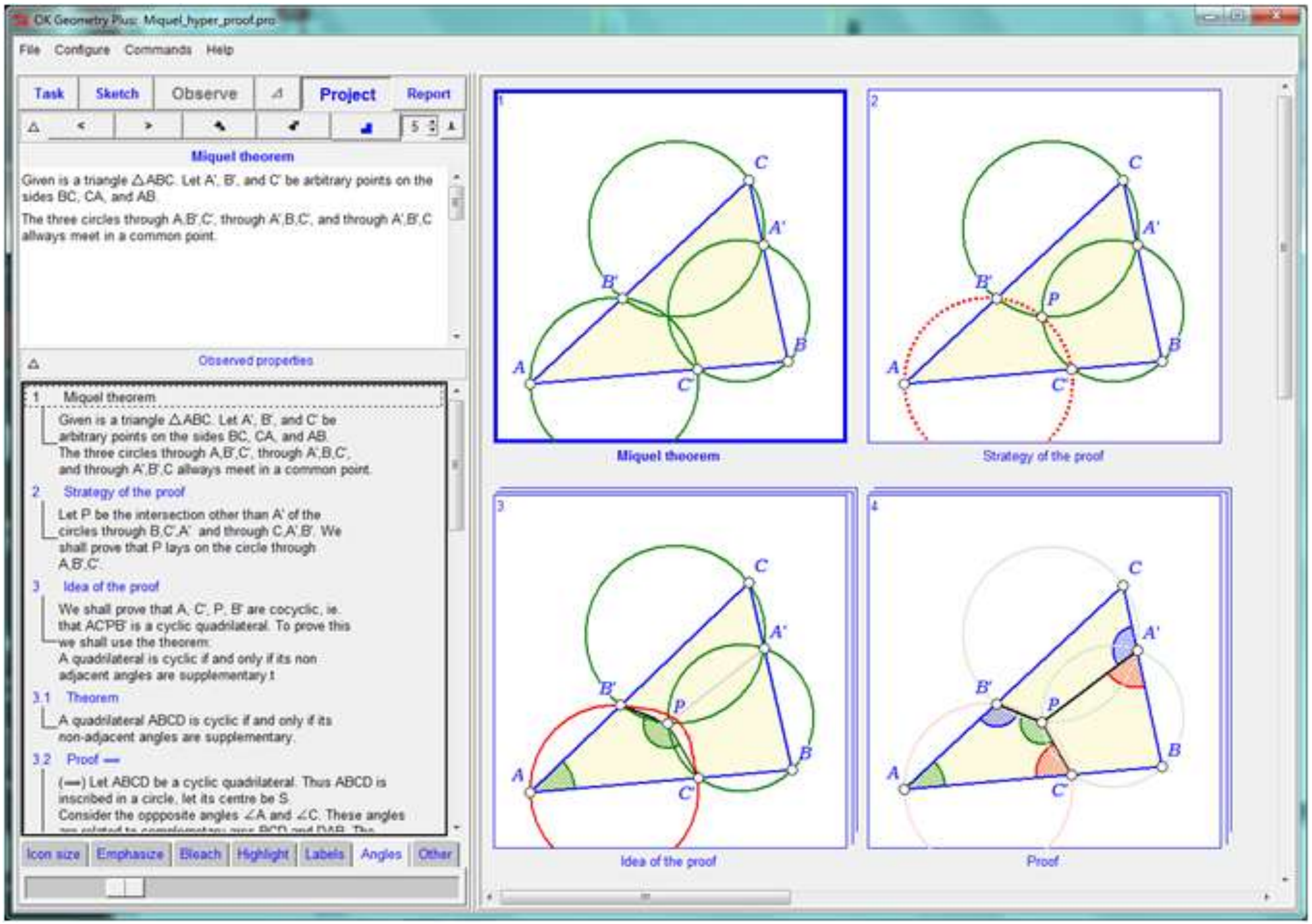
1. ABCD is a parallelogram (Given)
2. $\because$ ABCD is a parallelogram, $\therefore$ $\overline{AB} \,//\, \overline{DC}$ (Def. of parallelogram)
3. $\because$ ABCD is a parallelogram, $\therefore$ $\overline{AD} \,//\, \overline{BC}$ (Def. of parallelogram)
4. $\because \overline{AB} // \overline{DC}$, $\therefore \angle ACD = \angle CAB$ (Alt. int. anlges)
5. $\because \overline{AD} // \overline{BC}$, $\therefore \angle DAC = \angle BCA$ (Alt. int. anlges)
6. $\overline{AC} = \overline{AC}$ (Reflexive law)
7. $\Delta ABC \cong \Delta CDA$ (ASA)



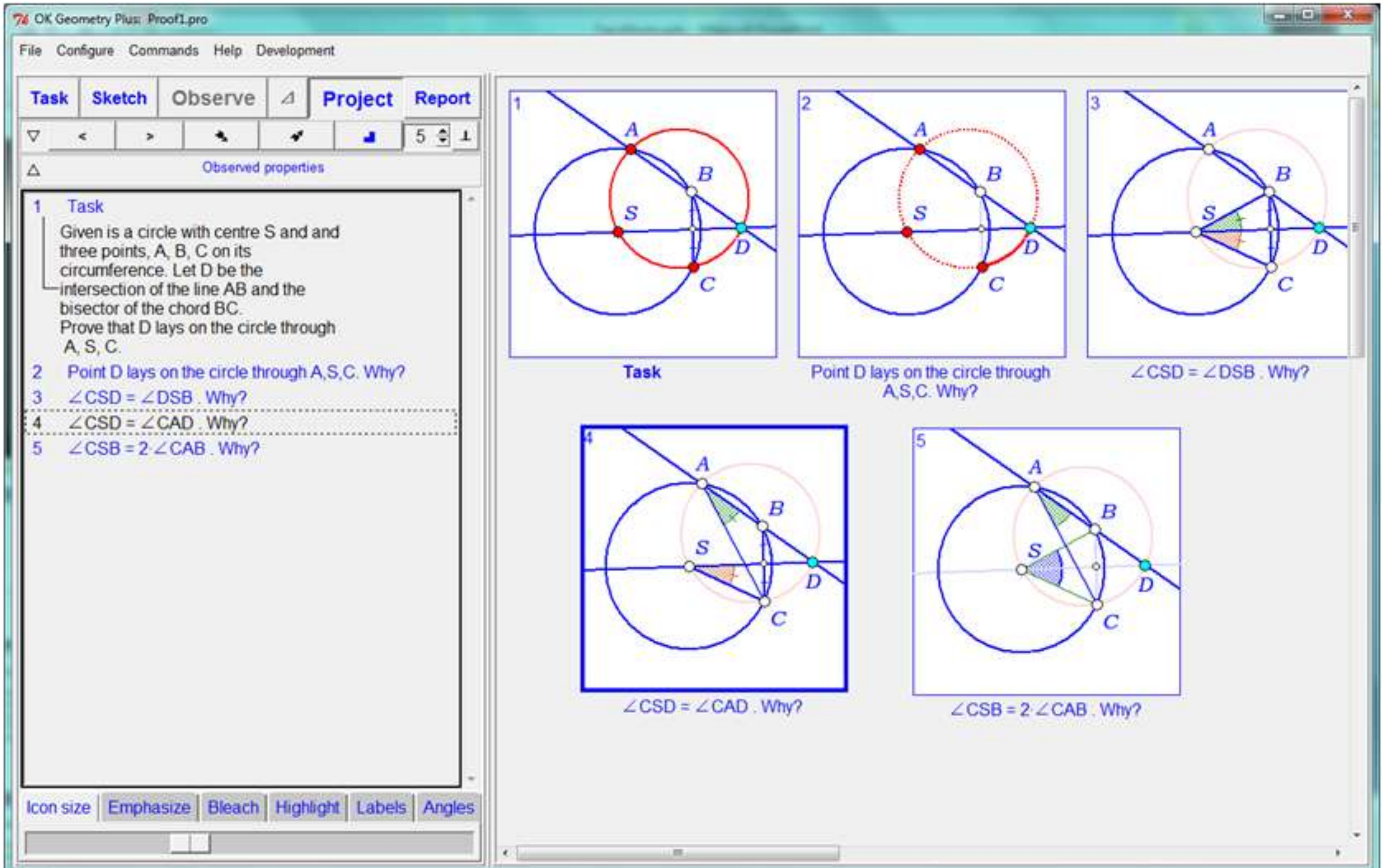
Justification of claims



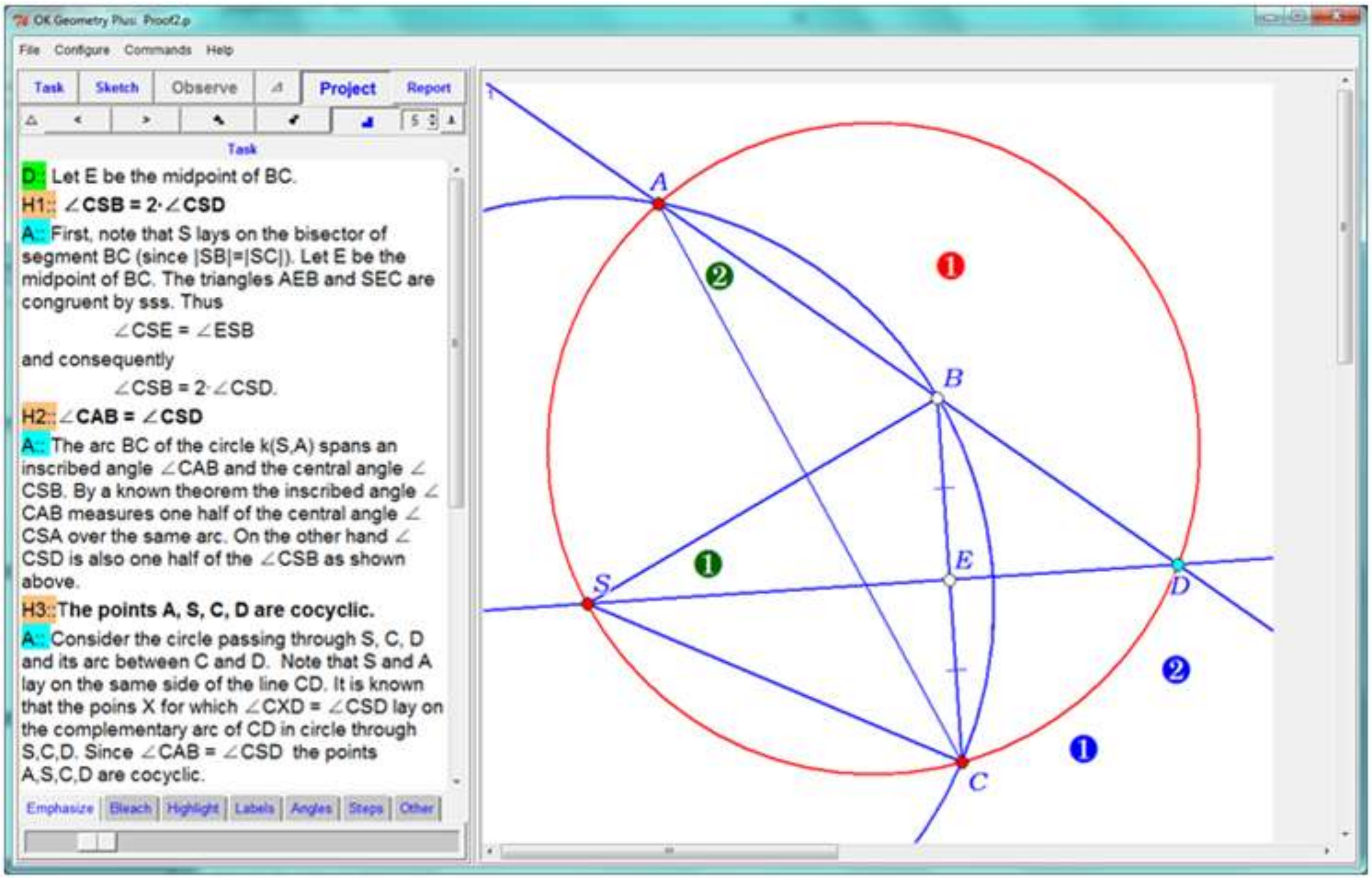
High-level ideas



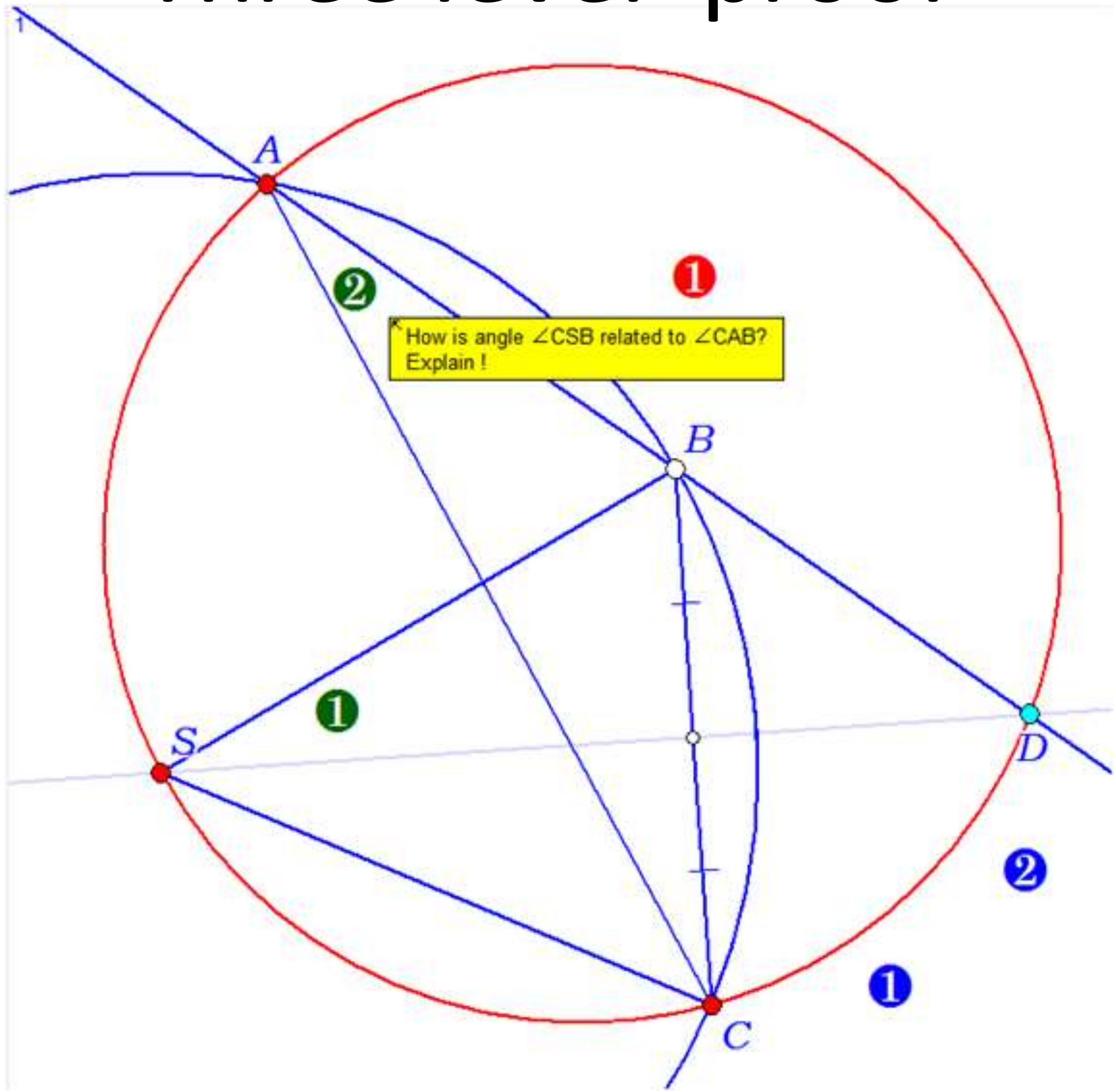
Chaining elements



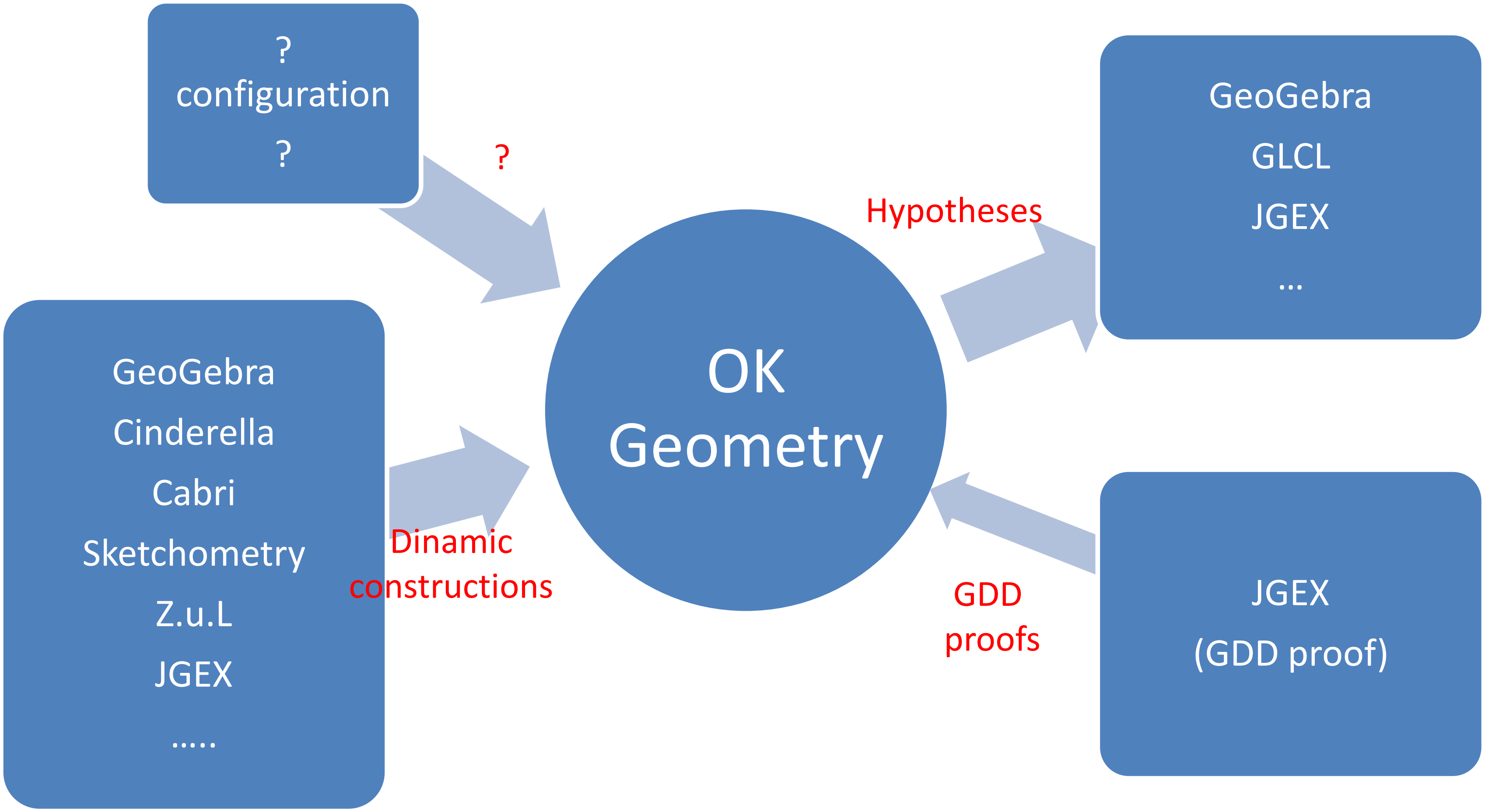
A D H statements



‘Three level’ proof



Observation and (A)DG tools



Thanks