14th International Conference on Automated Deduction in Geometry

Book of Abstracts

Pedro Quaresma & Zoltán Kovács

Belgrade, Serbia, 20–22 September 2023

Welcome Address

This is the *book of abstracts* of the 14th International Conference on Automated Deduction in Geometry (ADG 2023).

ADG is a forum to exchange ideas and views, to present research results and progress, and to demonstrate software tools at the intersection between geometry and automated deduction. The conference is held every two years. The previous editions of ADG were held in Hagenberg in 2021 (online, postponed from 2020 due to COVID-19), Nanning in 2018, Strasbourg in 2016, Coimbra in 2014, Edinburgh in 2012, Munich in 2010, Shanghai in 2008, Pontevedra in 2006, Gainesville in 2004, Hagenberg in 2002, Zurich in 2000, Beijing in 1998, and Toulouse in 1996.

This year ADG 2023 includes the workshop *Deduction in Education* on the 20th of September. In total there are, three Invited Talks, one Guest Lecture and 21 regular talks.

The 14th edition, ADG 2023, is held in Belgrade, Serbia, 20–22 of September, 2023.

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22 of September – Hotel Palace

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Programme

Invited Talks

20 Sept, 9:45 Julien Narboux, University of Strasbourg, France: Formalisation, arithmetization and automatisation of geometry

20 Sept, 10:45 Zlatan Magajna, University of Ljubljana, Slovenia: OK Geometry

21 Sept, 9:30 Filip Marić, University of Belgrade, Serbia: Automatization, formalization and visualization of hyperbolic geometry

Guest Lecture, 21 Sept, 17:30 Žarko Mijajlović, Application of Quantifier Elimination in Geometry

Book of Abstracts

Solving with GeoGebra Discovery an Austrian Mathematics Olympiad problem: lessons learned

20 Sept
11:30
Hotel Palace

Belén Ariño-Morera[1], Zoltán Kovács[2], Tomás Recio[3] and Piedad Tolmos[1]

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We address, through the automated reasoning tools in GeoGebra Discovery, a problem from a regional phase of the Austrian Mathematics Olympiad 2023. Trying to solve this problem gives rise to four different kind of feedbacks: the almost instantaneous, automated solution of the proposed problem; the measure of its complexity, according to some recent proposal; the automated discovery of a generalization of the given assertion, showing that the same statement is true over more general polygons than those mentioned in the problem; and the difficulties associated to the analysis of the surprising and involved high number of degenerate cases that appear when using the LocusEquation command in this problem. In our communication we will describe and reflect on these diverse issues, enhancing its exemplar role for showing some of the advantages, problems, and current fields of development of GeoGebra Discovery.

Keywords: Automated theorem proving • Automated deduction in geometry • GeoGebra.

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20 Sept
12:00
Hotel Palace

Solving some geometry problems of the Náboj 2023 contest with automated deduction in GeoGebra Discovery

Amela Hota[1], Zoltán Kovács[1] and Alexander Vujic[1]

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In this article, we solve some of the geometry problems of the Náboj 2023 competition with the help of a computer, using examples that the software tool GeoGebra Discovery can calculate. In each case, the calculation requires symbolic computations. We analyze the difficulty of feeding the problem into the machine and set further goals to make the problems of this type of contests even more tractable in the future.

Keywords: Náboj competition • GeoGebra Discovery • Automated deduction in geometry • Mathematics education • Artificial Intelligence.

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20 Sept
14:00
Hotel Palace

The locus story of a rocking camel in a medical center in the city of Freistadt

Eva Erhart[1], Anna Käferböck[1], Zoltán Kovács[1] and Engelbert Zeintl[1]

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We give an example of automated geometry reasoning for an imaginary classroom project by using the free software package GeoGebra Discovery. The project is motivated by a publicly available toy, a rocking camel, installed at a medical center in Upper Austria. We explain how the process of a false conjecture, experimenting, modeling, a precise mathematical setup, and then a proof by automated reasoning could help extend mathematical knowledge at secondary school level and above.

Keywords: 4-Bar Linkages • Elimination • GeoGebra Discovery • Locus Equation.

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20 Sept
14:30
Hotel Palace

Using GXWeb for theorem proving and mathematical modelling

Danny Aley[1] and Philip Todd[1]

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GXWeb is the free browser based version of the symbolic geometry software Geometry Expressions. We demonstrate its use in an educational setting with examples from theorem proving, mathematical modelling and loci and envelopes.

Keywords: Automated Deduction Geometry • Geometric Theorem Discovery • Euclidean Geometry.

Using Java Geometry Expert as guide in the preparations for math contests

20 Sept
15:00
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Ines Ganglmayr[1], Zoltán Kovács[1]

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We give an insight into Java Geometry Expert (JGEx) in use in a school context, focusing on the Austrian school system. JGEx can offer great support in some classroom situations, especially for solving mathematical competition tasks. Also, we discuss some limitations of the program.

Keywords: mathematical competition tasks • Java Geometry Expert • structured thinking

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Showing proofs, assessing difficulty with GeoGebra Discovery

20 Sept
16:00
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Zoltán Kovács[1],Tomás Recio[2] and M. Pilar Vélez[2]

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In our contribution we describe some on-going improvements concerning the Automated Reasoning Tools developed in GeoGebra Discovery, providing different examples of the performance of these new features. We describe the new ShowProof command, that outputs both the sequence of the different steps performed by GeoGebra Discovery to confirm a certain statement, as well as a number pretending to grade the difficulty or interest of the assertion. The proposal of this assessment measure, involving the comparison of the expression of the thesis as a combination of the hypotheses, will be developed.

Keywords: Automated theorem proving • Automated deduction in geometry • GeoGebra.

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20 Sept
16:30
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3D space trajectories and beyond: abstract art creation with 3D printing

Thierry Dana-Picard[1], Mathias Tejera[2] and Eva Ulbrich[2]

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We present simple models of trajectories in space, both in 2D and in 3D. The first examples, which model bicircular moves in the same direction, are classical curves (epicycloids, etc.). Then, we explore bicircular moves in reverse direction and tricircular moves. The exploration is followed by 3D printing. Students' activities are organized around this exploration.

Keywords: Space curves • models of orbits • Animations • 3D printing.

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20 Sept
17:00
Hotel Palace

Deduction in Education: the Case of Serbia

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The main goal of the paper is to provide insight into the current state of education in Serbia regarding the use of educational aspects of software related to deduction, geometry and mathematical reasoning. Authors investigate to what extent and for what purposes teachers and students use such software. The consideration of the current state is substantiated by the examples of teaching practices at different school levels, and supplemented by the proposals which tend to contribute to deeper understanding of the topic. The most commonly used automatic deduction and calculation programs in Serbian education are GeoGebra, Photomath, Microsoft Math Solver, Desmos, and Wolfram Alpha. These mathematical software represent an advantageous topic for cross-curricular connection in schools which could be realized in many different ways. It is suggested in the context of connecting mathematics and physics to art, history of science, national history of science, philosophy, logic and psychology.

Keywords: Cross-curricular Connection • GeoGebra • Education.

Euclid's theorems through the area method

20 Sept
17:30
Hotel Palace

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All secondary school curricula include the theory of similar figures. Book VI of Euclid's Elements is the historical source of this theory. It includes the intercept theorem (VI.2), the criteria for similarity of triangles (VI.4–7) and the theorem that similar figures are to each other as the square of the similarity scale (VI.19–20). In the paper, we prove these theorems through the Area Method, which provides axioms for that part of the Elements, and then using the program GCLC which implements the Area Method.

Keywords: Euclidean proportion • similar figures • area method • prover GCLC.

Open source prover in the attic

21 Sept
11:00
Hotel Palace

Zoltán Kovács[1] and Alexander Vujic[1]

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The well known JGEx program became open source a few years ago, but seemingly, further development of the program can only be done without the original authors. In our project, we are looking at whether it is possible to continue such a large project as a newcomer without the involvement of the original authors. Is there a way to internationalize, fix bugs, improve the code base, add new features? In other words, to save a relic found in the attic and polish it into a useful everyday tool.

Keywords: JGEx • Java development • Automated Deduction in Education.

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21 Sept
11:30
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Automation of Triangle Ruler-and-Compass Constructions Using Constraint Solvers

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In this paper, we present an approach to automated solving of triangle ruler-and-compass construction problems using finite-domain constraint solvers. The constraint model is described in the MiniZinc modeling language, and is based on the automated planning. The main benefit of using general constraint solvers for such purpose, instead of developing dedicated tools, is that we can rely on the efficient search that is already implemented within the solver, enabling us to focus on geometric aspects of the problem. We may also use the solver's built-in optimization capabilities to search for the shortest possible constructions. We evaluate our approach on 74 solvable problems from the Wernick's list, and compare it to the dedicated triangle construction solver ArgoTriCS. The results show that our approach is comparable to dedicated tools, while it requires much less effort to implement. Also, our model often finds shorter constructions, thanks to the optimization capabilities offered by the constraint solvers.

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Automated Completion of Statements and Proofs in Synthetic Geometry: an Approach based on Constraint Solving

Salwa Tabet Gonzalez[1], Predrag Janičić[2] and Julien Narboux[1]

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Conjecturing and theorem proving are activities at the center of mathematical practice and are difficult to separate. In this paper, we propose a framework for completing incomplete conjectures and incomplete proofs. The framework can turn a conjecture with missing assumptions and with an under-specified goal into a proper theorem. Also, the proposed framework can help in completing a proof sketch into a human-readable and machine-checkable proof. Our approach is focused on synthetic geometry, and uses coherent logic and constraint solving. The proposed approach is uniform for all three kinds of tasks, flexible and, to our knowledge, unique such approach.

Keywords: synthetic geometry • automated deduction • proofs • constraint solving • abducts.

Towards an Independent Version of Tarski's System of Geometry

21 Sept
14:00
Hotel Palace

Pierre Boutry[1], Stéphane Kastenbaum[2] and Clément Saintier[2]
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In 1926–1927, Tarski designed a set of axioms for Euclidean geometry which reached its final form in a manuscript by Schwabhäuser, Szmielew and Tarski in 1983. The differences amount to simplifications obtained by Tarski and Gupta. Gupta presented an independent version of Tarski's system of geometry, thus establishing that his version could not be further simplified without modifying the axioms. To obtain the independence of one of his axioms, namely Pasch's axiom, he proved the independence of one of its consequence: the previously eliminated symmetry of betweenness. However, an independence model for the non-degenerate part of Pasch's axiom was provided by Szczerba for another version of Tarski's system of geometry in which the symmetry of betweenness holds. This independence proof cannot be directly used for Gupta's version as the statements of the parallel postulate difier. In this paper, we present our progress towards obtaining an independent version of a variant of Gupta's system. Compared to Gupta's version, we split Pasch's axiom into this previously eliminated axiom and its non-degenerate part and change the statement of the parallel postulate. We verified the independence properties by mechanizing counter-models using the Coq proof-assistant.

Keywords: Independence • Coq • Formalization of geometry.

Towards automated readable proofs of ruler and compass constructions

21 Sept
14:30
Hotel Palace

Vesna Marinković[1], Tijana Šukilović[1] and Filip Marić[1]
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Although there are several systems that successfully generate construction steps for ruler and compass construction problems, none of them provides readable synthetic correctness proofs for generated constructions. In the present work, we demonstrate how our triangle construction solver ArgoTriCS can cooperate with automated theorem provers for first order logic and coherent logic so that it generates construction correctness proofs, that are both human-readable and formal (can be checked by interactive theorem provers such as Coq or Isabelle/HOL). These proofs currently rely on many high-level lemmas and our goal is to have them all formally shown from the basic axioms of geometry.

Keywords: triangle construction problems • coherent logic • automated theorem proving.

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9:30
Hotel Palace

Considerations on Approaches and Metrics in Automated Theorem Generation/Finding in Geometry

Pedro Quaresma[1], Pierluigi Graziani[2] and Stefano M. Nicoletti[3]

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The pursue of what are properties that can be identified to permit an automated reasoning program to generate and find new and interesting theorems is an interesting research goal (pun intended). The automatic discovery of new theorems is a goal in itself, and it has been addressed in specific areas, with different methods. The separation of the "weeds", uninteresting, trivial facts, from the "wheat", new and interesting facts, is much harder, but is also being addressed by different authors using different approaches. In this paper we will focus on geometry. We present and discuss different approaches for the automatic discovery of geometric theorems (and properties), and different metrics to find the interesting theorems among all those that were generated. After this description we will introduce the first result of this article: an undecidability result proving that having an algorithmic procedure that decides for every possible Turing Machine that produces theorems, whether it is able to produce also interesting theorems, is an undecidable problem. Consequently, we will argue that judging whether a theorem prover is able to produce interesting theorems remains a non deterministic task, at best a task to be addressed by program based in an algorithm guided by heuristics criteria. Therefore, as a human, to satisfy this task two things are necessary: an expert survey that sheds light on what a theorem prover/finder of interesting geometric theorems is, and—to enable this analysis—other surveys that clarify metrics and approaches related to the interestingness of geometric theorems. In the conclusion of this article we will introduce the structure of two of these surveys—the second result of this article—and we will discuss some future work.

Keywords: Interesting Problems • Wos' Problem 31 • automated theorem proving.

Theorem Discovery amongst Cyclic Polygons

22 Sept
10:00
Hotel Palace

Philip Todd[1]

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We examine a class of geometric theorems on cyclic 2n-gons. We prove that if we take n disjoint pairs of sides, each pair separated by an even number of polygon sides, then there is a linear combination of the angles between those sides which is constant. We present a formula for the linear combination, which provides a theorem statement in terms of those angles. We describe a program which uses this result to generate new geometry proof problems and their solutions.

Keywords: Automated Deduction Geometry • Geometric Theorem Discovery • Euclidean Geometry.

Automatic Transformations of Coq Proof Scripts—Work in Progress

22 Sept
11:00
Hotel Palace

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Proof assistants like Coq are increasingly popular to help mathematicians carry out proofs of the results they conjecture. However, formal proofs remain highly technical and are especially difficult to reuse. In this paper, we present a framework to carry out a posteriori script transformations. These transformations are meant to be applied as an automated post-processing step, once the proof has been completed. As an example, we present a transformation which takes an arbitray large proof script and produces an equivalent single-line proof script, which can be executed by Coq in one single step. Other applications, such as fully expanding a proof script (for debugging purposes), removing all named hypotheses, etc. could be developped within this framework. We apply our tool to various Coq proof scripts, including some from the GeoCoq library.

Keywords: Coq • proof transformation • GeoCoq.

Geometrically Analyzing the Equilibria of Parametric Biochemical Networks Admitting Linear Conservation Laws: Case Studies

22 Sept
11:30
Hotel Palace

Changbo Chen[1] and Wenyuan Wu[1]

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In a recent work, we introduced an efficient geometric method, called GeoBlock, for analyzing the equilibria of bi-parametric biological systems by exploiting the block structure. This approach requires that the system defining the equilibria is square and is zero-dimensional for generically chosen parameters. In this paper, by introducing a bipartite graph to describe the correlations between variables and constraints, we show how to adapt GeoBlock to efficiently handle biochemical networks admitting linear conservation laws. Two biochemical networks, with respective 11 and 16 variables, are used to illustrate the effectiveness of the approach. The experiments show that the presented geometric approach is a good complementary to the existing algebraic elimination method or the grid sampling approach for solving such systems.

Improving Angular Speed Uniformity by Piecewise Radical Reparameterization

22 Sept
12:00
Hotel Palace

Hoon Hong[1], Dongming Wang[2,3] and Jing Yang[2]

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In this paper, we show how to reparameterize any rational parametric curve whose angular speed function has zeros by means of radical piecewise transformation, such that the resulting reparameterization has nonzero uniform angular speed. The reparameterization algorithm we propose uses different strategies for the optimization of the parameters in the transformation to improve the uniformity. An example is provided to illustrate the performance of the proposed algorithm. This work extends our previous study on the problem of reparameterization for rational curves with nonzero angular speed and can be used to enhance the quality of curve plotting when the angular speed of the given parametric curve may vanish.

Keywords: Parameterization • Angular speed uniformity • Reparameterization • Radical transformation • Piecewise Möbius transformation.

The Companion and Bézout Subresultants of Two Bernstein Polynomials

22 Sept
14:00
Hotel Palace

Mei Tan[1] and Jing Yang[1]

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In this paper, we provide two methods for computing the subresultant polynomials of Bernstein polynomials, i.e., one using companion subresultant matrix and the other using Bézout subresultant matrix. It should be pointed out that the input polynomials are given in Bernstein form, and so are the output subresultant polynomials. Thus the benefit of the new methods is that the coefficient structure of the given Bernstein polynomials is maintained and one may directly compute the subresultant polynomials without converting the polynomials between the Bernstein form and the power-basis form.

Keywords: Resultant • Bézout subresultant • Bernstein polynomial • Companion subresultant.

Automated proof of Ramsey theorem via symbolic computation

22 Sept
14:30
Hotel Palace

Jian Lu[1], Zhenbing Zeng[1] and Liangyu Chen[2]

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This paper presents an algebraic method for automatically proving Ramsey's theorem and demonstrates the automatic proof of $R(3,3) = 6$ and $R(3,4) = 9$ using symbolic computation. Using this method we have also proved an improvement of Ramsey theorem, which is to say that for the complete graph of six vertices whose edges are arbitrarily colored by two colors, there exist at least two monochromatic triangles.

Keywords: Ramsey theorem • Machine proof • Symbolic computation • Complete graph • Combinatorial geometry.

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