Theorem Discovery amongst Cyclic Polygons

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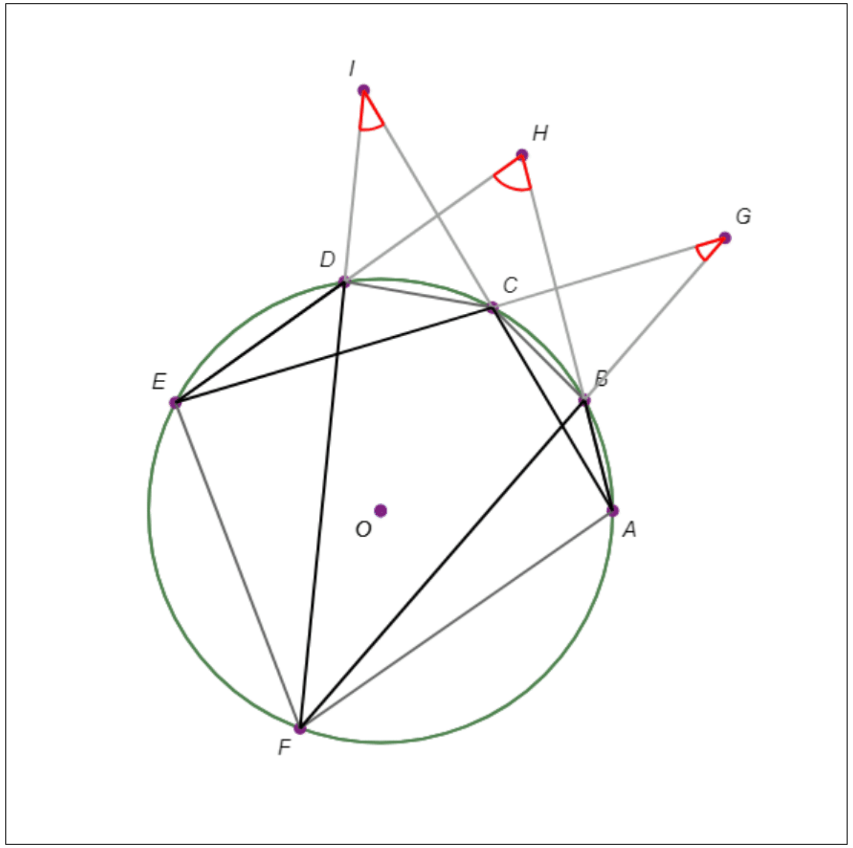
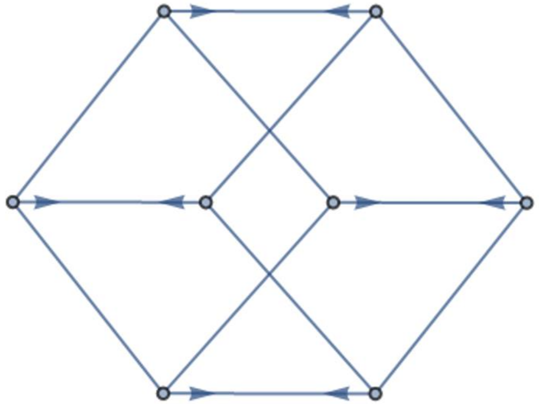
Problem Generator



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of CE and FB. Let H be the intersection of ED and BA. Let I be the intersection of DF and AC. Angle DHB = x. Angle CGB = y. Find angle DIC. .
(difficulty 22)

Answer | New Problem | Create Collection | Add to Custom Collection

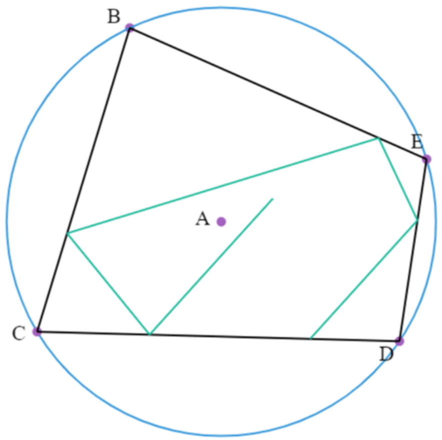
Automated Discovery of Angle Theorems

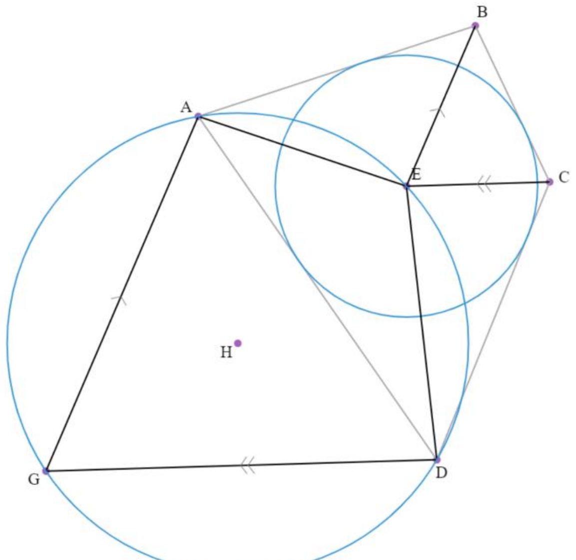


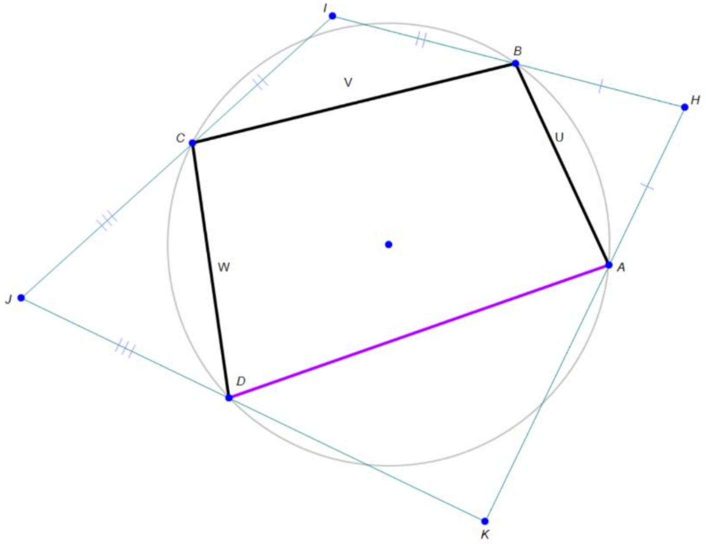
Theorems with angle bisectors, reflections, circle chords

Graph theoretical characterization, geometrical interpretation

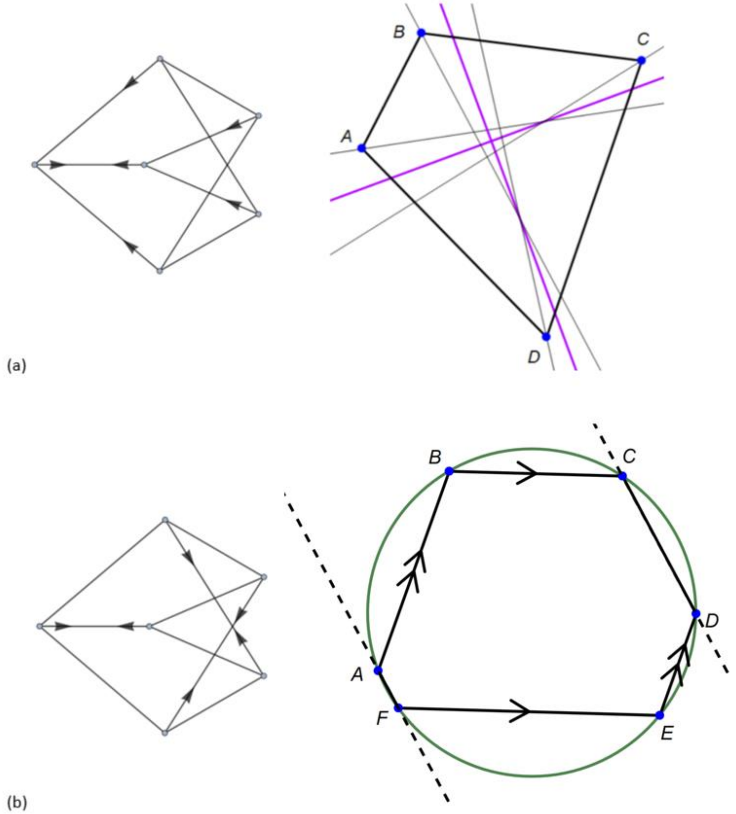
Multiple geometrical representations of a single graph







Automated Discovery of Angle Theorems



Theorems with angle bisectors, reflections, circle chords

Graph theoretical characterization, geometrical interpretation

Multiple geometrical representations of a single graph

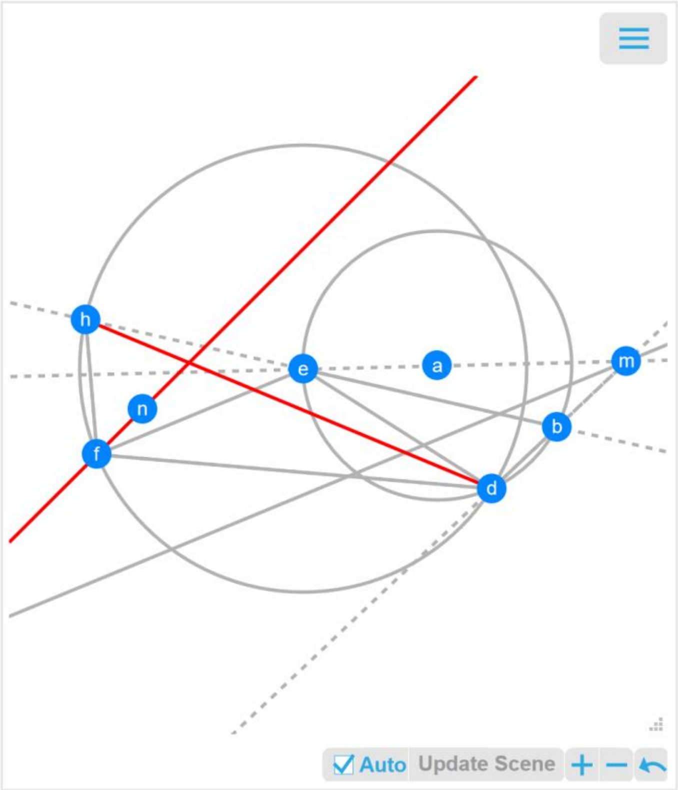
Cyclic polygons have clean diagrams as radial lines need not be explicit

A Program to Create new Geometry Proof Problems

Automatically creates proof problems (theorems) from randomly chosen graphs.

Uses Mathematica GeometricScene

Biased in favor of cyclic polygons

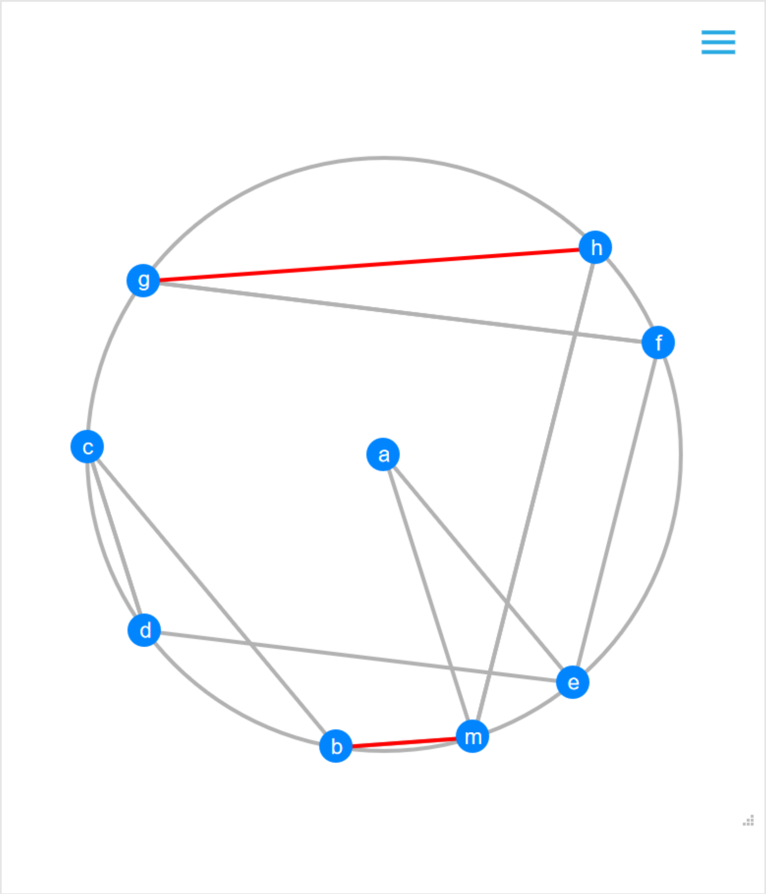


Let bed be a triangle with circumcentre a. Let fdh be a triangle with circumcentre e. Let ebh be collinear. Let L1 be the angle bisector of dfh. Let L2 be the angle bisector of bd and ae. Let ef be parallel to L2. Prove L1 is 67.5 degrees to dh.

A Program to Create new Geometry Proof Problems

Some problems involve single cyclic polygons and parallel pairs.

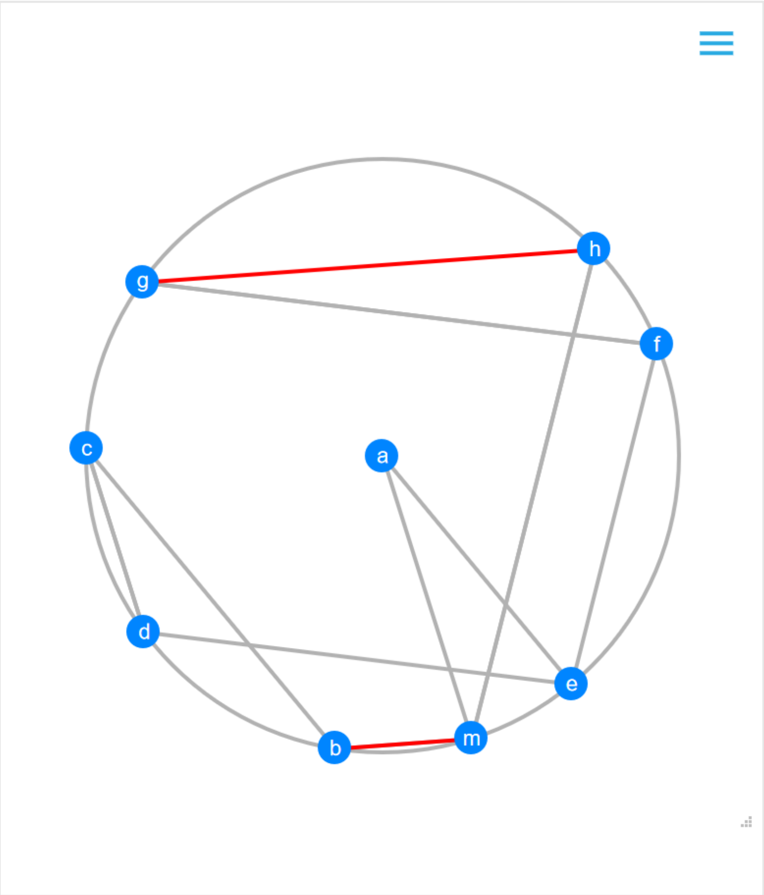
We note that these parallel pairs could be replaced by specified non-zero angles.

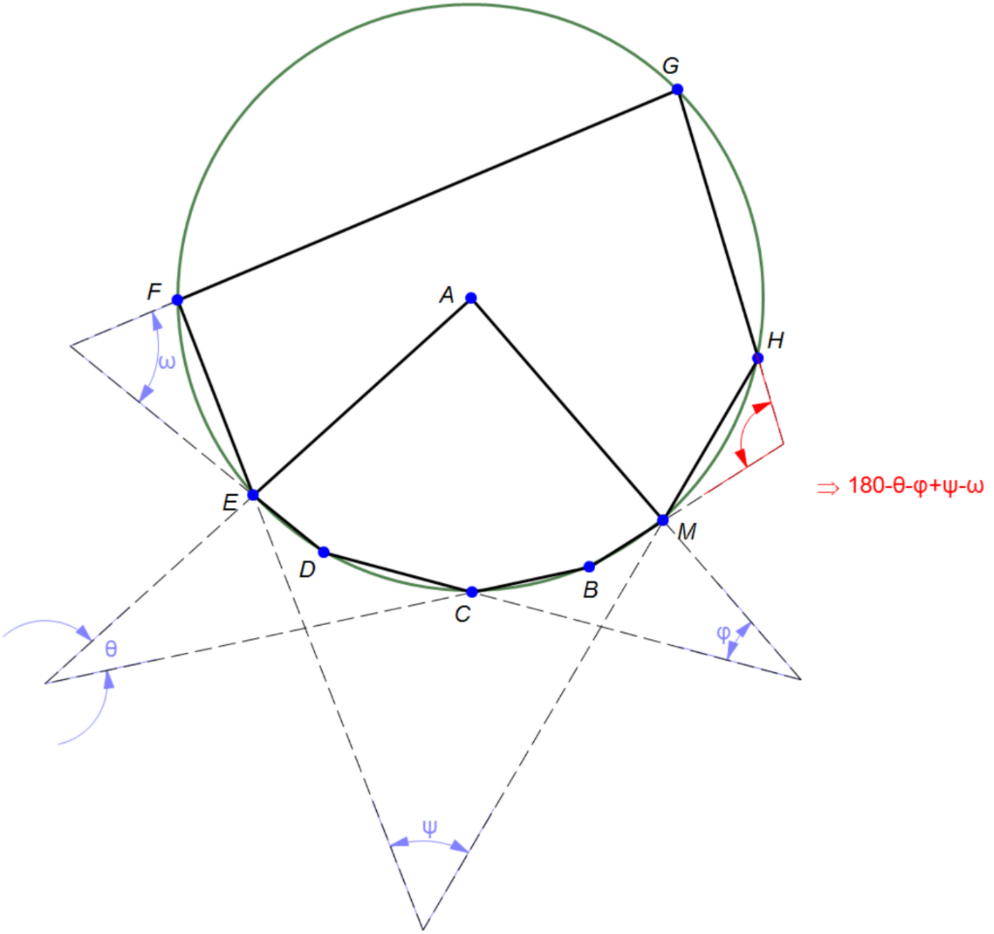


Let bcdefghm be a cyclic octagon with centre a. Let ae be parallel to bc. Let am be parallel to dc. Let de be parallel to gf. Let ef be parallel to mh. Prove bm is parallel to gh.

Replace parallelism by specified angles

(cleaner diagram)



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Let bcdefghm be a cyclic octagon with centre a. Let ae be parallel to bc. Let am be
  parallel to dc. Let de be parallel to gf. Let ef be parallel to mh. Prove bm is paralle
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Theorem Discovery amongst Cyclic Polygons

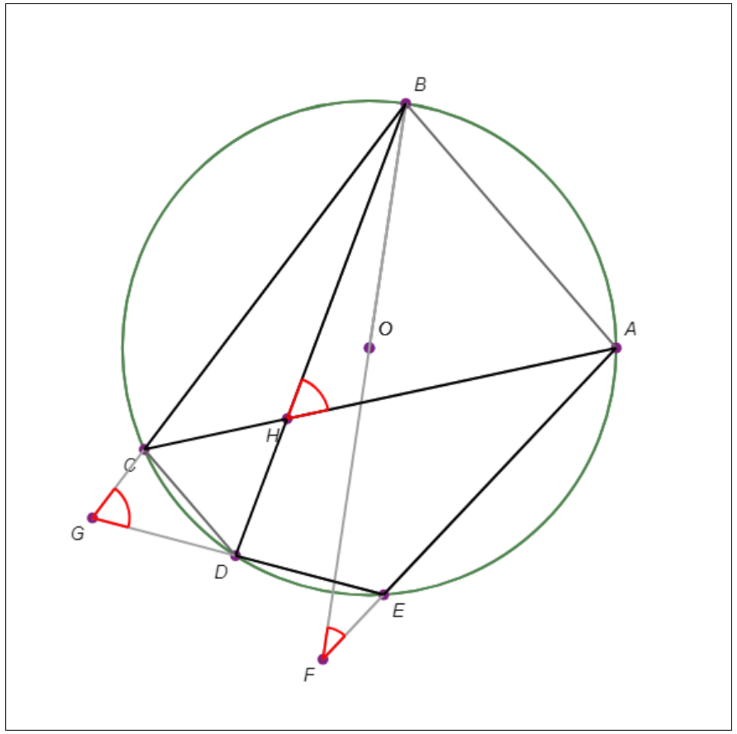
In this presentation, we limit ourselves to theorems involving a single cyclic polygon (perhaps self-intersecting)

However, we allow non-zero angles in the premise and conclusion

This provides a simple context to illustrate the general approach

We develop the method from first principles

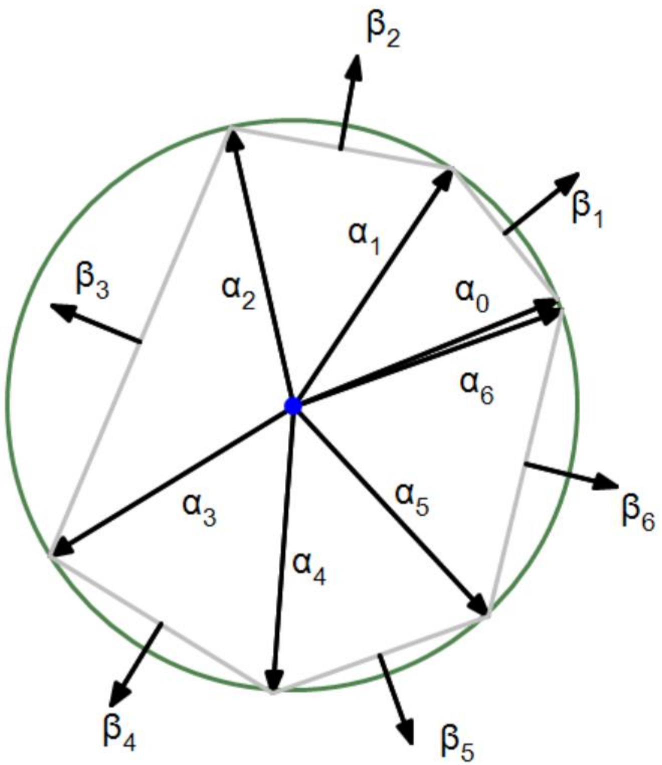
Problem Generator



- ▶ Problem Settings
- ▶ Collection Settings
- ▶ Custom Collection

Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of OB and AE. Let G be the intersection of BC and ED. Let H be the intersection of CA and DB. Angle BFE = x. Angle CGD = y. Find angle AHB. .
(difficulty 27)

Answer | New Problem | Create Collection | Add to Custom Collection



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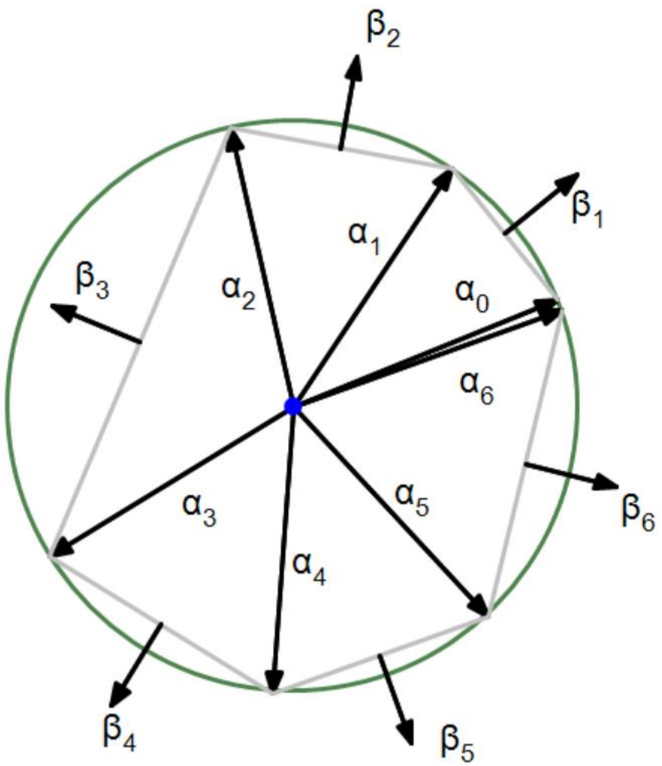

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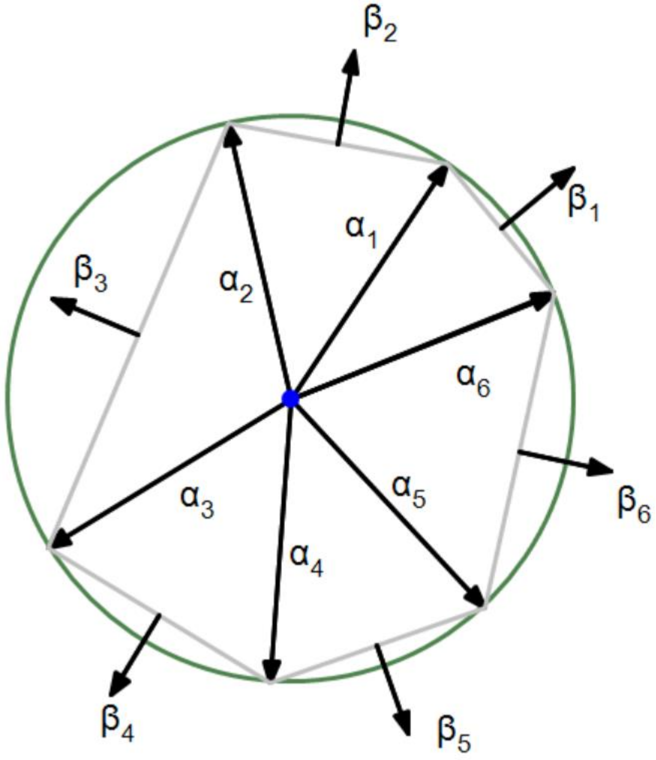

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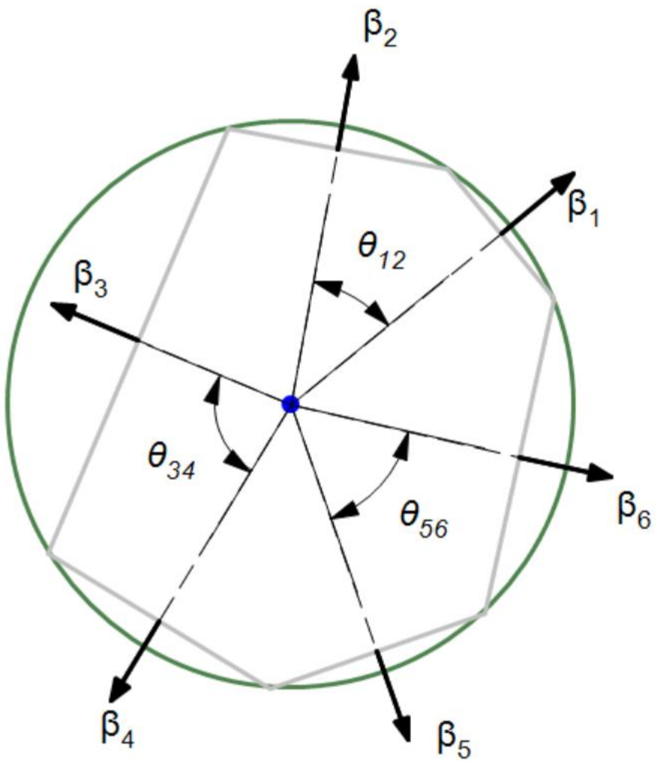

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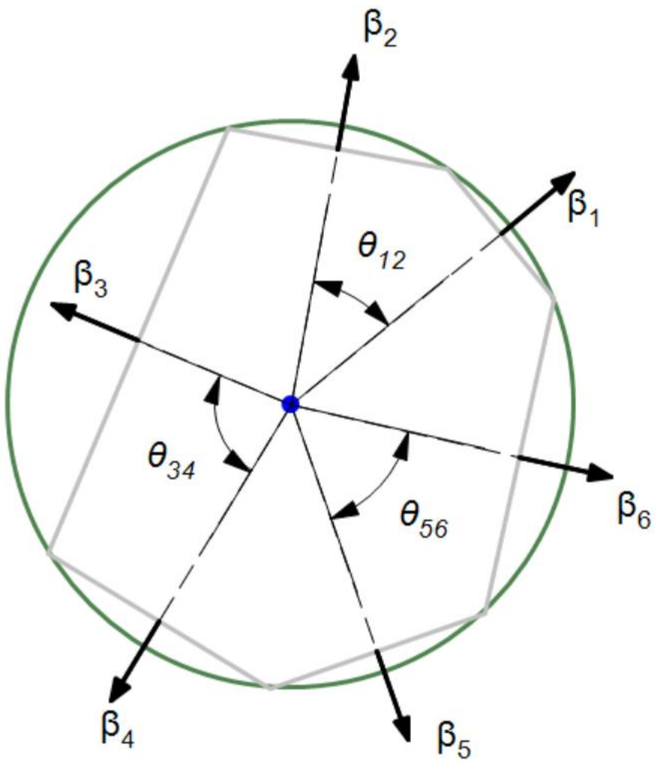
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & -2 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & -2 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1
\end{pmatrix}
\begin{pmatrix}
\alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6
\end{pmatrix}
=
\begin{pmatrix}
2\pi \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \theta_{12} \\ \theta_{34} \\ \theta_{56}
\end{pmatrix}
$$



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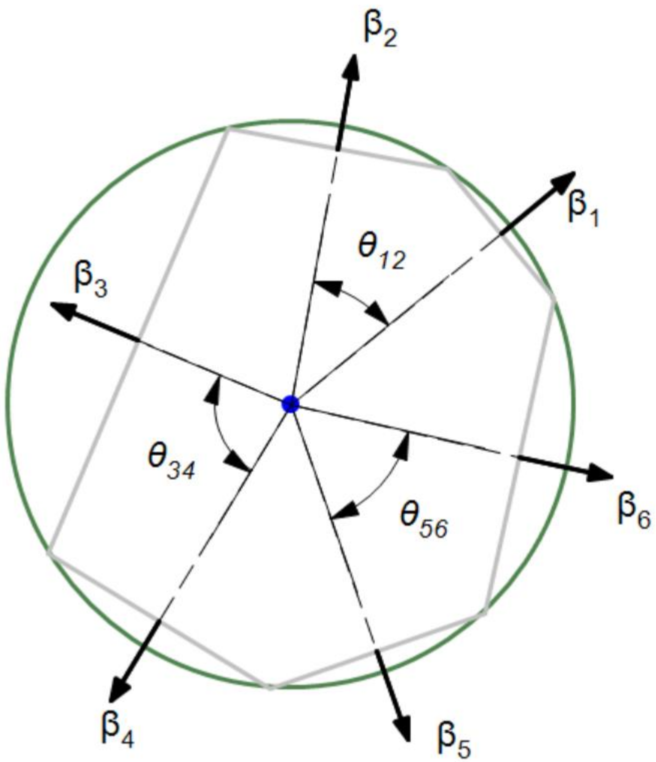

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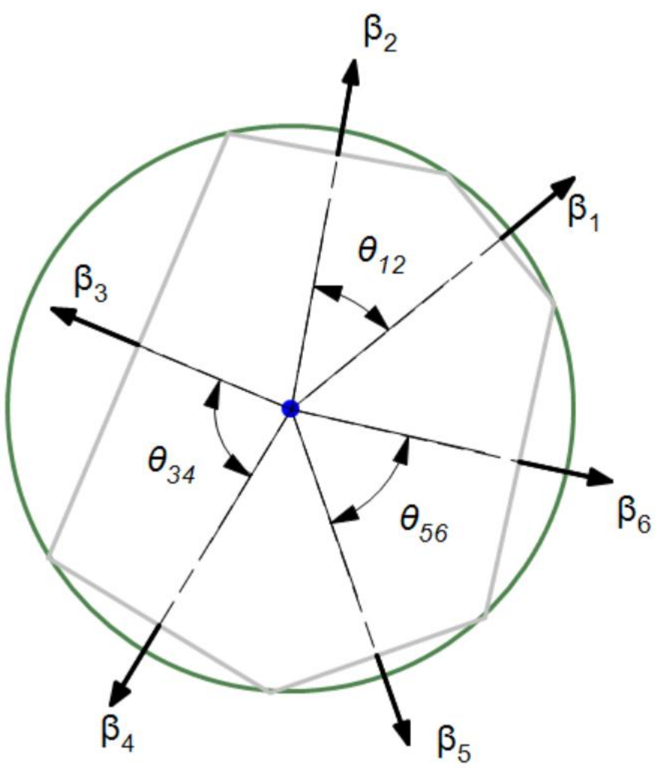

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0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & -2 & 0
\end{pmatrix}
\begin{pmatrix}
\alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6
\end{pmatrix}
=
\begin{pmatrix}
2\pi \\ 2\theta_{12} \\ 0 \\ 2\theta_{34} \\ 0 \\ 2\theta_{56}
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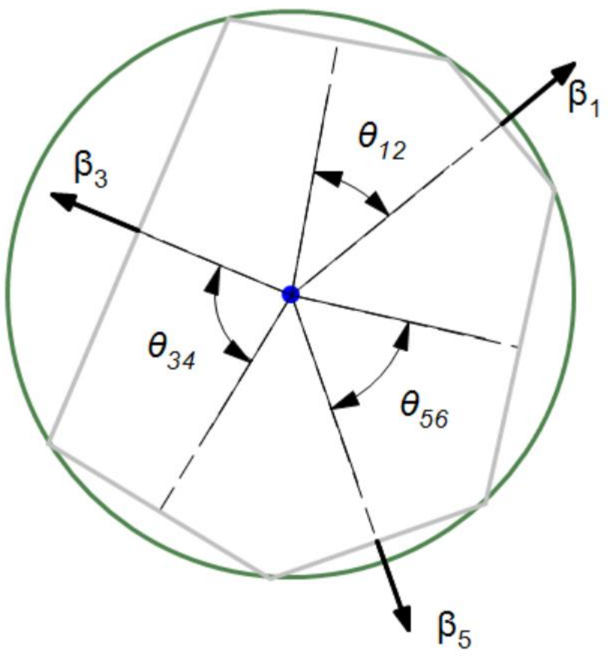

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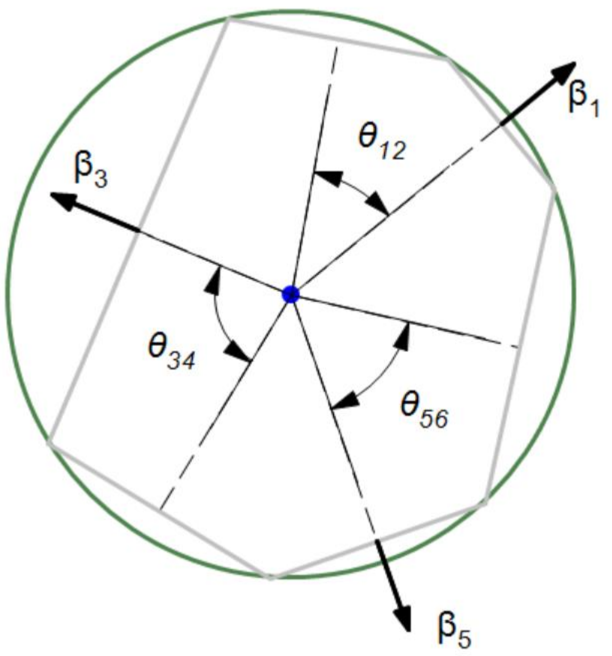

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0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & -2
\end{pmatrix}
\begin{pmatrix}
\alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \\ \beta_1 \\ \beta_3 \\ \beta_5
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=
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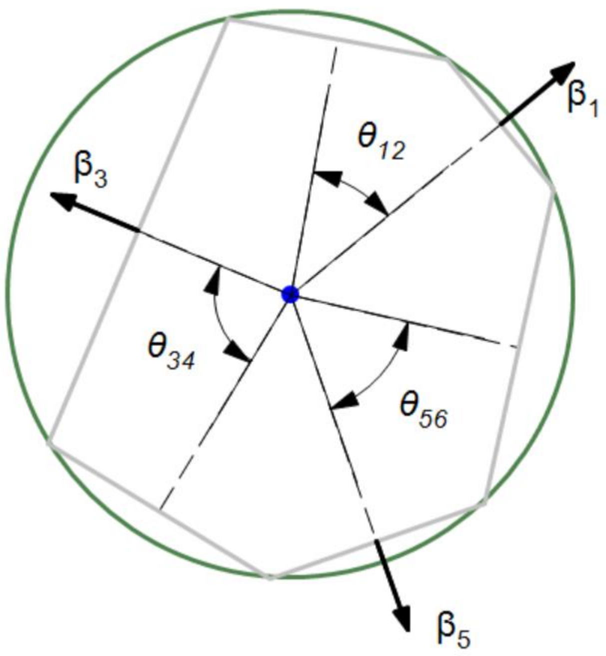
$$



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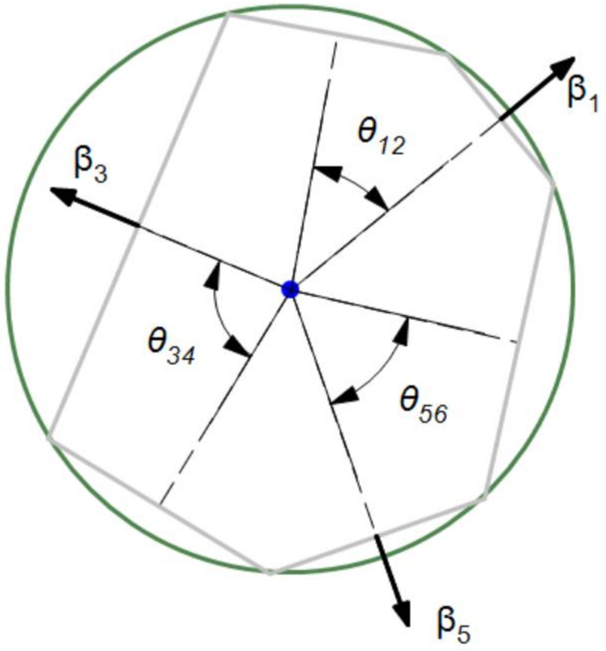

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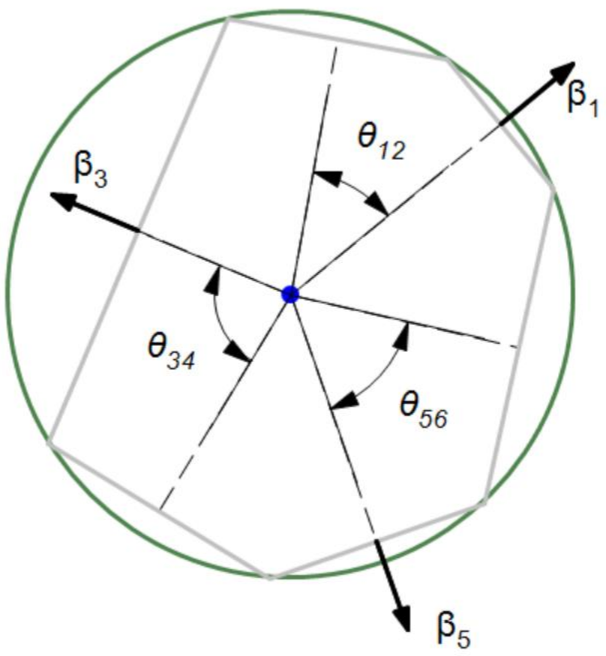

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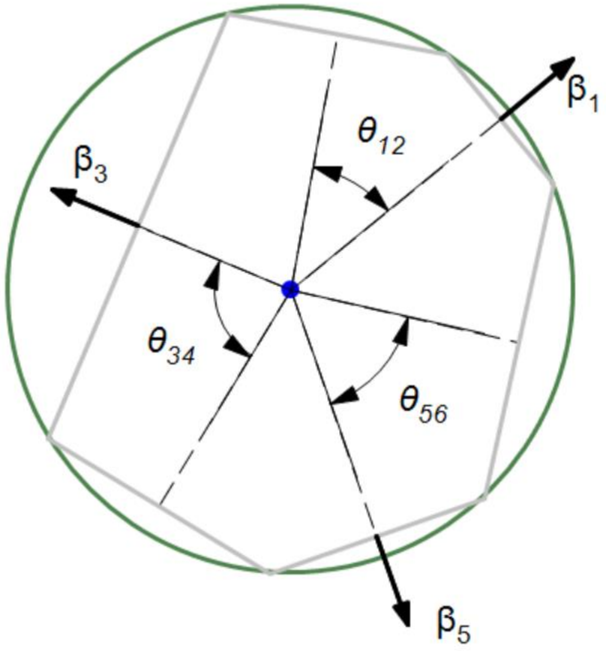

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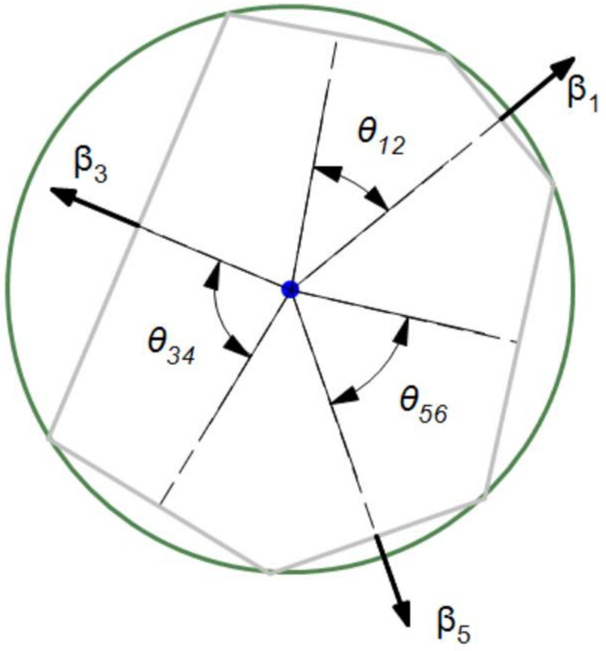

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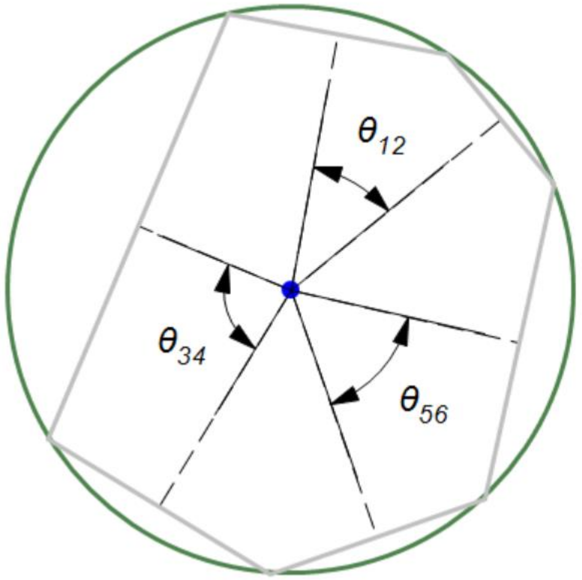
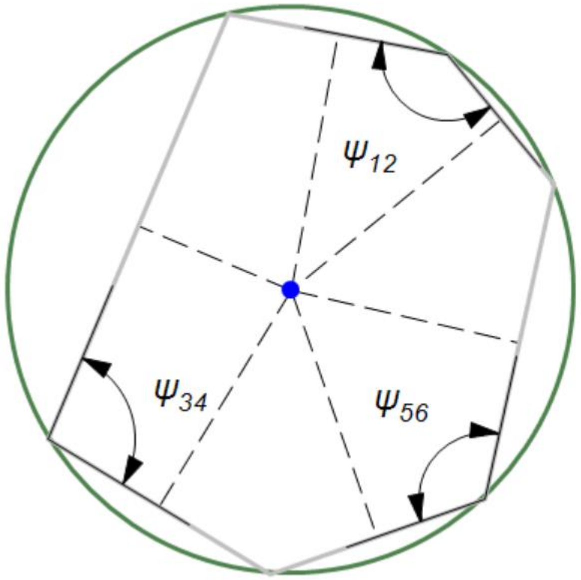
$$\theta_{56} + \theta_{34} + \theta_{12} = \pi$$





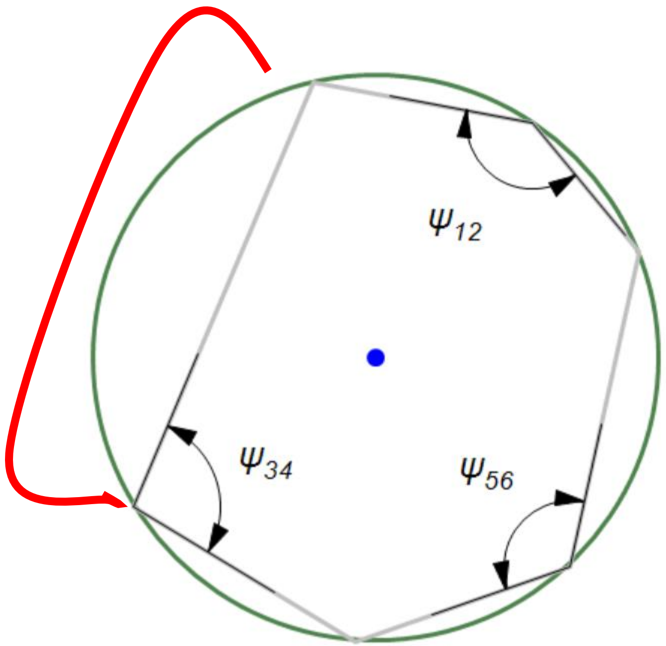
$$\theta_{56} + \theta_{34} + \theta_{12} = \pi$$

$$\psi_{56} + \psi_{34} + \psi_{12} = 2\pi$$

Many theorems from one

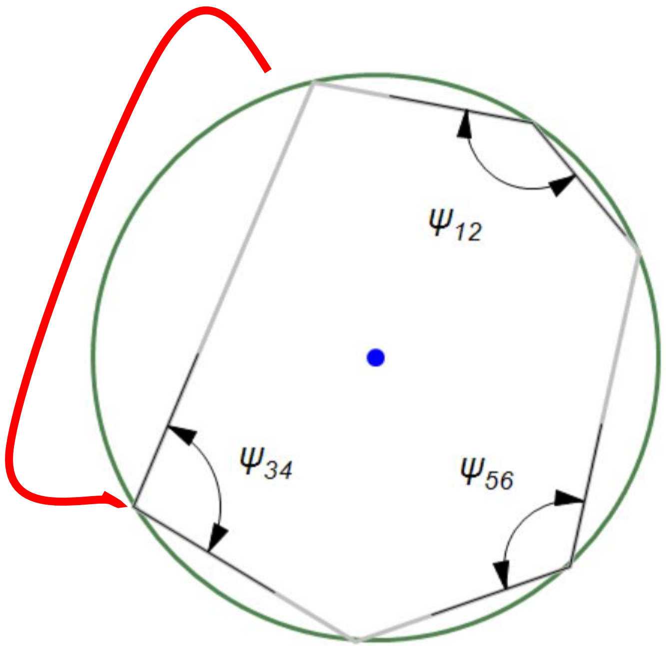
- Permute vertices
- Merge Vertices

Permute vertices

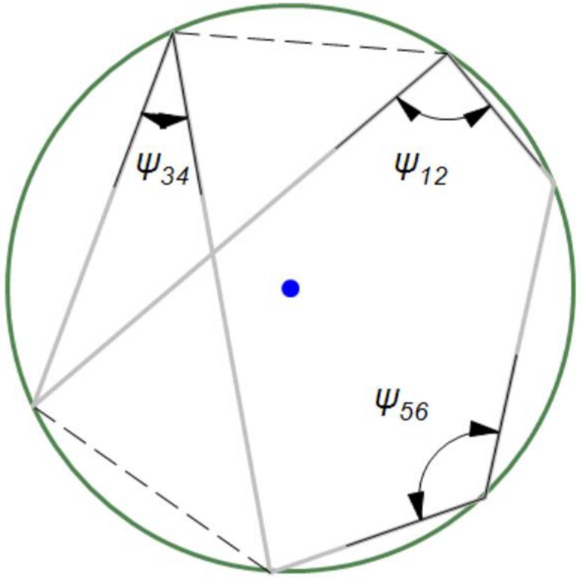


$$\psi_{56} + \psi_{34} + \psi_{12} = 2\pi$$

Permute vertices

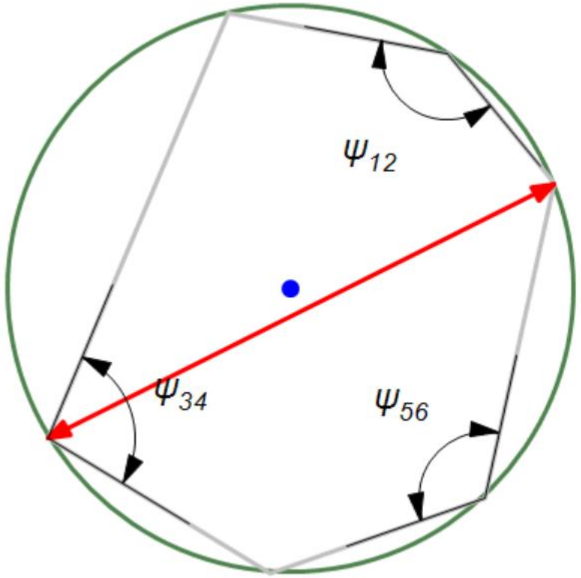


$$\psi_{56} + \psi_{34} + \psi_{12} = 2\pi$$



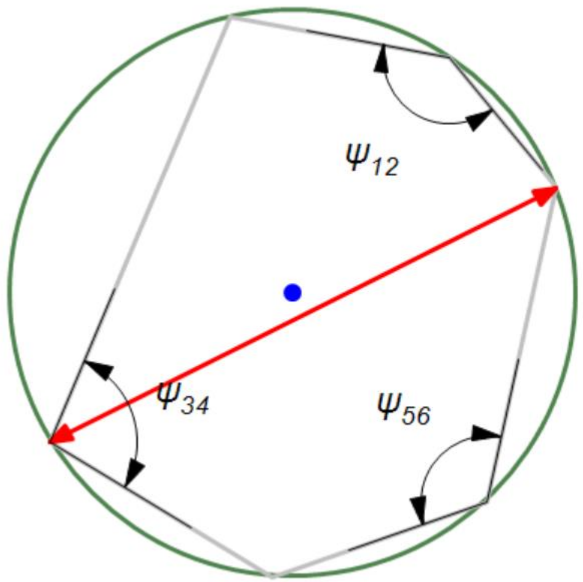
$$\psi_{56} - \psi_{34} + \psi_{12} = \pi$$

Merge Vertices

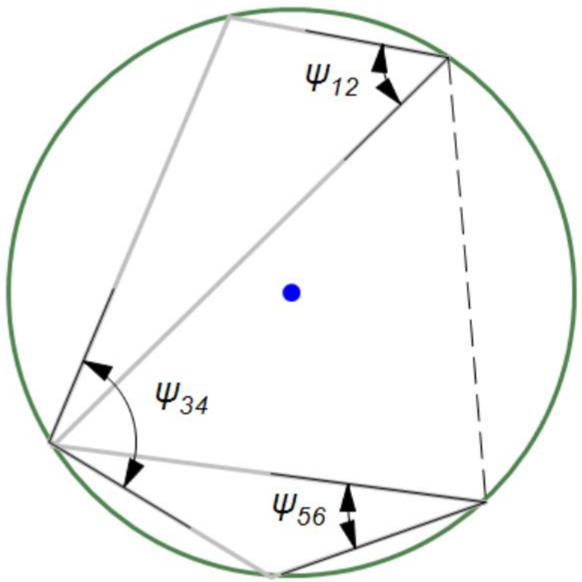


$$\psi_{56} + \psi_{34} + \psi_{12} = 2\pi$$

Merge Vertices

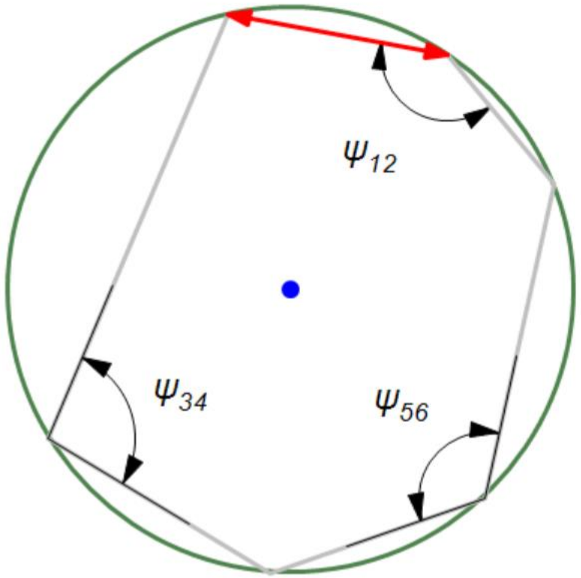


$$\psi_{56} + \psi_{34} + \psi_{12} = 2\pi$$



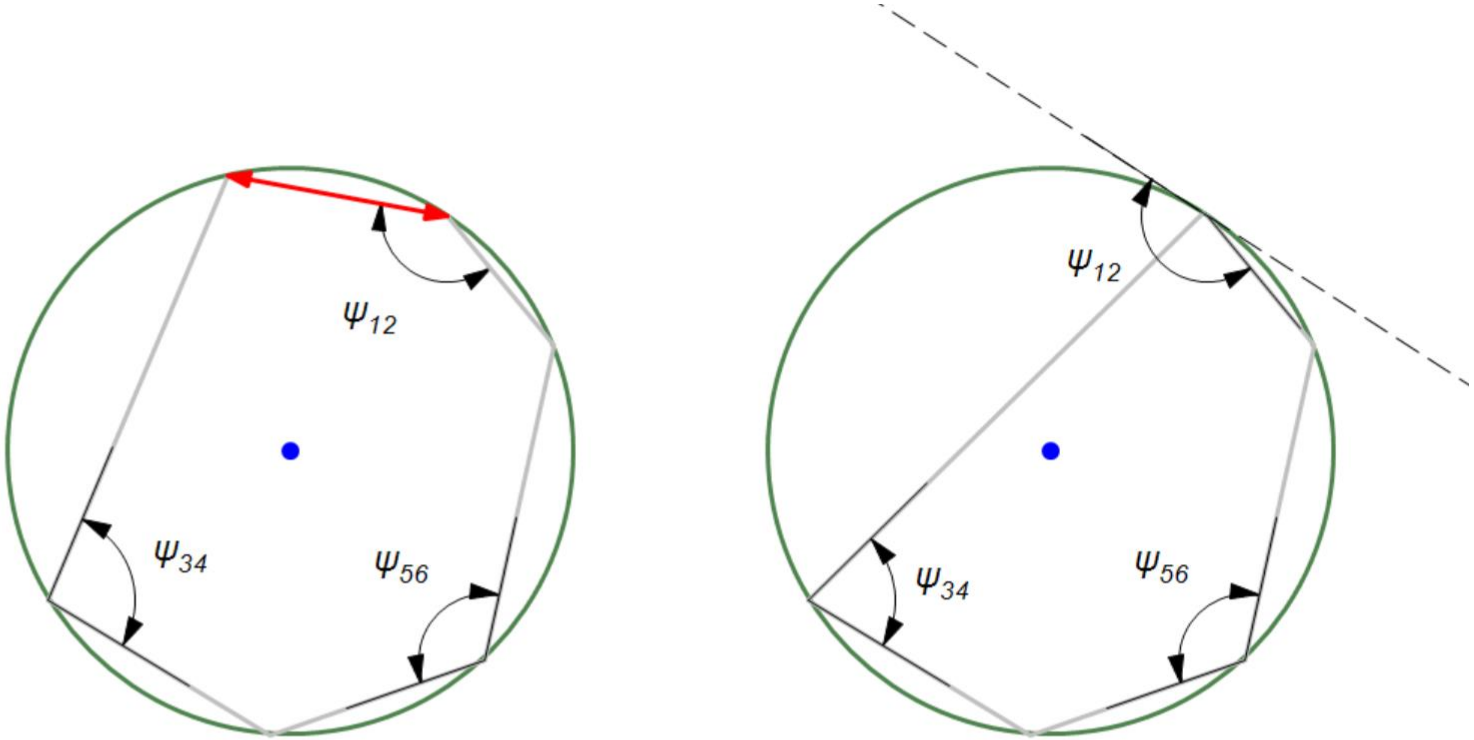
$$\psi_{56} + \psi_{34} + \psi_{12} = \pi$$

Merge Vertices



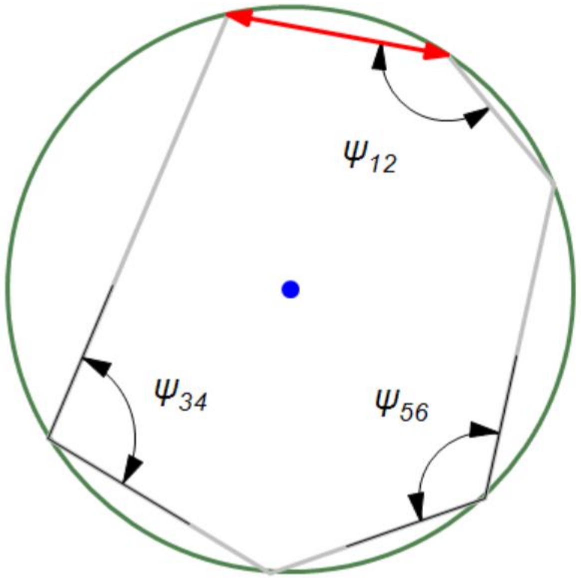
$$\psi_{56} + \psi_{34} + \psi_{12} = 2\pi$$

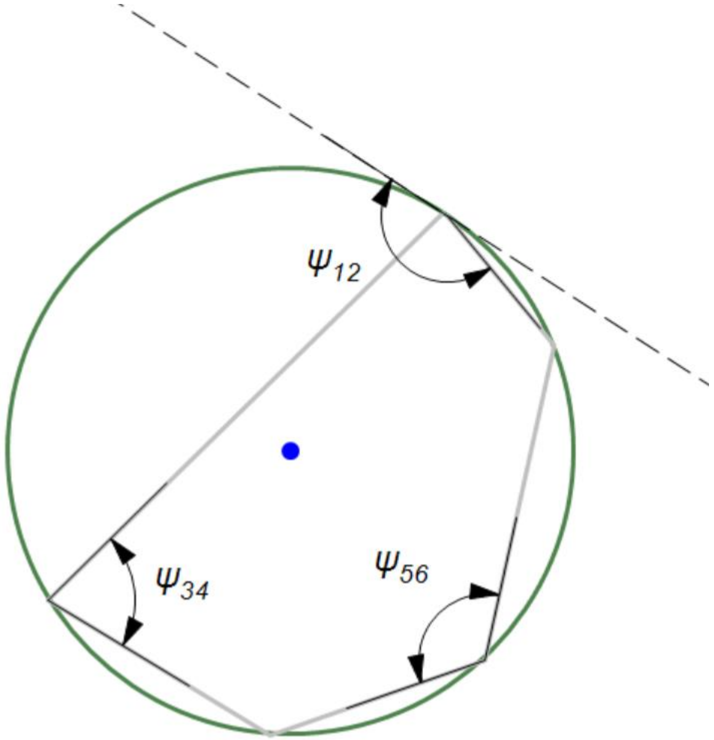
Merge Vertices

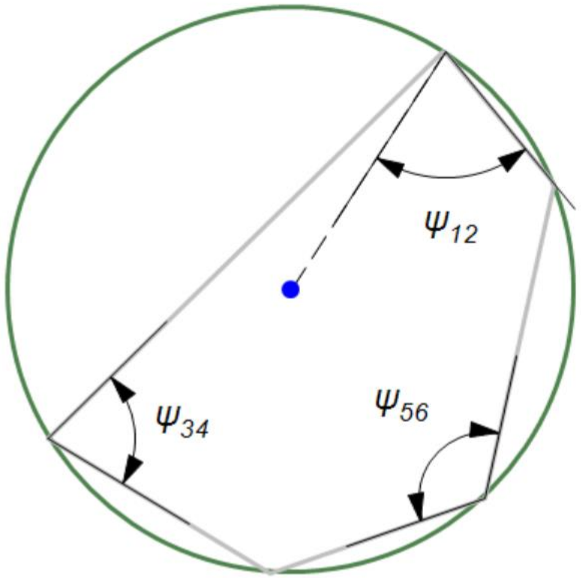


$$\psi_{56} + \psi_{34} + \psi_{12} = 2\pi$$

Merge Vertices



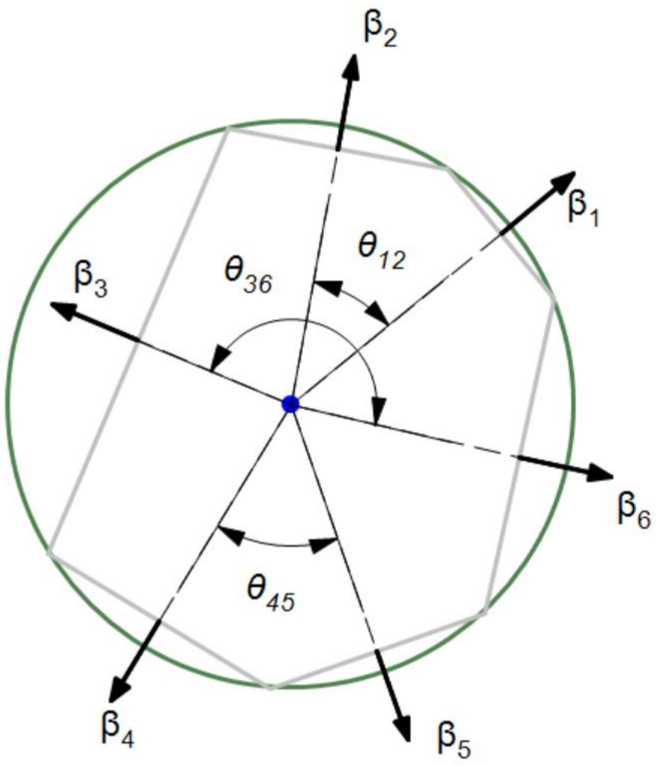




$$\psi_{56} + \psi_{34} + \psi_{12} = 2\pi$$

$$\psi_{56} + \psi_{34} + \psi_{12} = \frac{3\pi}{2}$$

Another Angle Combination



$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & -2 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & -2 & 0 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \\ \beta_1 \\ \beta_3 \\ \beta_5 \end{pmatrix} = \begin{pmatrix} 2\pi \\ 2\theta_{12} \\ 0 \\ 2\theta_{36} \\ 0 \\ 2\theta_{45} \end{pmatrix}$$

Another Angle Combination

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & -2 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & -2 & 0 \end{pmatrix}$$

Another Angle Combination

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & -2 & 0 \end{pmatrix}$$

Another Angle Combination

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & -2 & 0 \end{pmatrix}$$

Another Angle Combination

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 2 & -2 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & -2 & 0 \end{pmatrix}$$

Another Angle Combination

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 2 & -2 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & -2 & 0 \end{pmatrix}$$

Another Angle Combination

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 2 & -2 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

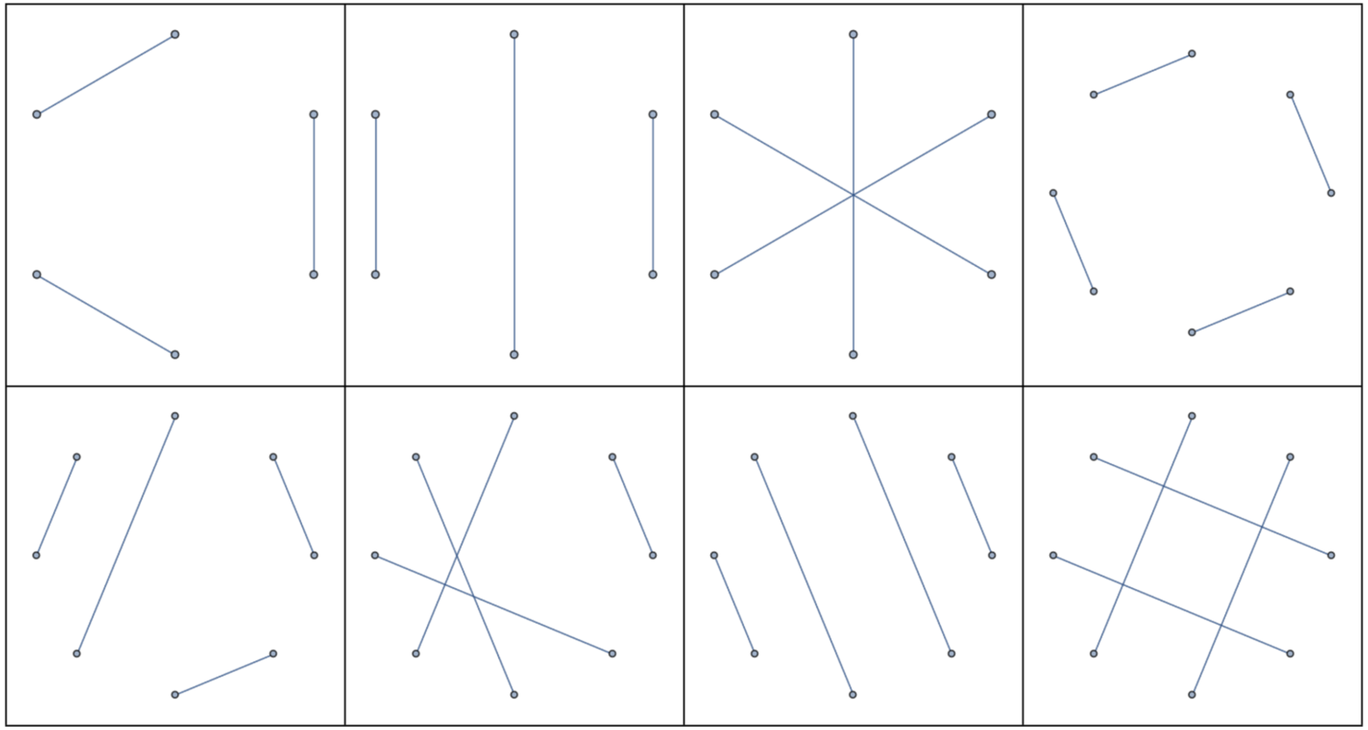
Another Angle Combination

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 2 & -2 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & -2 & 0 \end{pmatrix}$$

Must be separated by an even number of rows

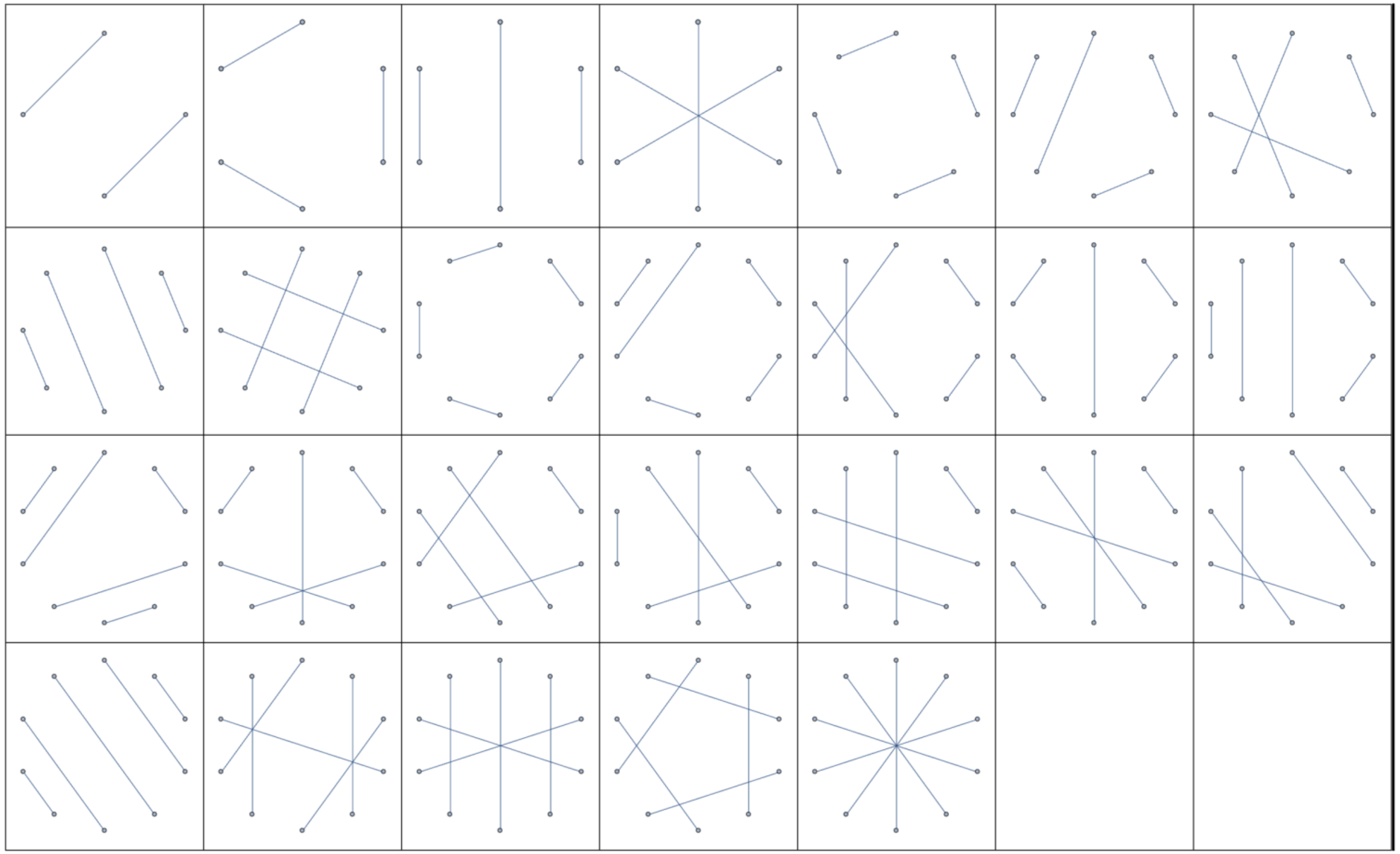
How many patterns are there?

n	patterns
6	3
8	5



How many patterns are there?

n	patterns
4	1
6	3
8	5
10	17



How many patterns are there?

n	patterns
4	1
6	3
8	5
10	17
12	53
14	260

The OEIS is supported by the many generous donors to the OEIS Foundation.

THE ON-LINE ENCYCLOPEDIA OF INTEGER SEQUENCES ®

founded in 1964 by N. J. A. Sloane

a357442 Search Hints

(Greetings from The On-Line Encyclopedia of Integer Sequences!)

Search: **a357442**

Displaying 1-1 of 1 result found. page 1

Sort: relevance | references | number | modified | created Format: long | short | data

A357442 Consider a clock face with 2*n "hours" marked around the dial; a(n) = number of ways to match the even hours to the odd hours, modulo rotations and reflections. +40 1

1, 1, 3, 5, 17, 53, 260, 1466, 10915, 93196, 917898, 10015299, 119914982, 1557364352, 21797494987, 326930305166, 5230756117008, 88922108947567, 1600594738591550, 30411281088326498, 608225534389576956, 12772735698577492558 (list; graph; refs; listen; history; text; internal format)

OFFSET 1,3

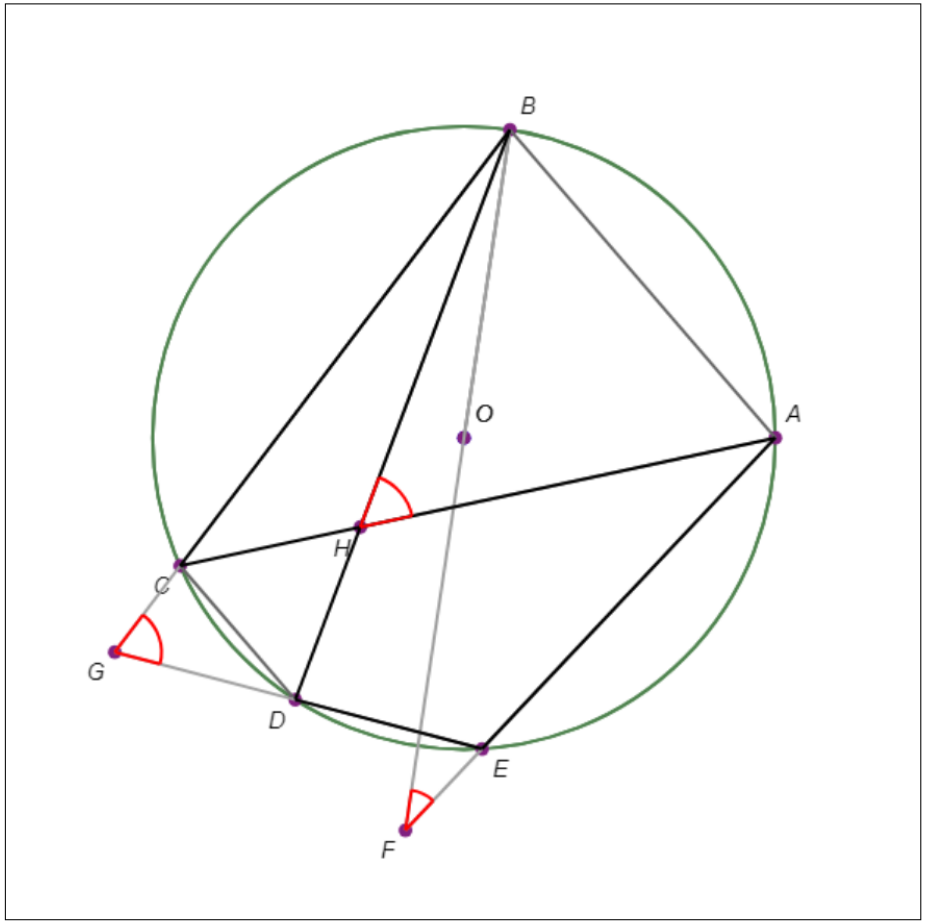
LINKS Table of n, a(n) for n=1..22.
Barry Cipra, Illustration for a(5) = 17
N. J. A. Sloane, Sketch illustrating a(1) = a(2) = 1, a(3) = 3

Problem Generator

1. Choose angle pattern
2. Select vertex permutation
3. Choose vertices to merge
4. Find good instance of the problem
5. Create step by step solution
6. Estimate difficulty of problem

www.saltire.com/ProblemGenerator

Problem Generator



Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of OB and AE. Let G be the intersection of BC and ED. Let H be the intersection of CA and DB. Angle BFE = x. Angle CGD = y. Find angle AHB. .
(difficulty 27)

Answer | New Problem | Create Collection | Add to Custom Collection

Good Problem Instance

1. None of the angles too close to 0 or 180
2. Intersection points not too far outside the circle
3. No lines look like they pass through a point unless they really do

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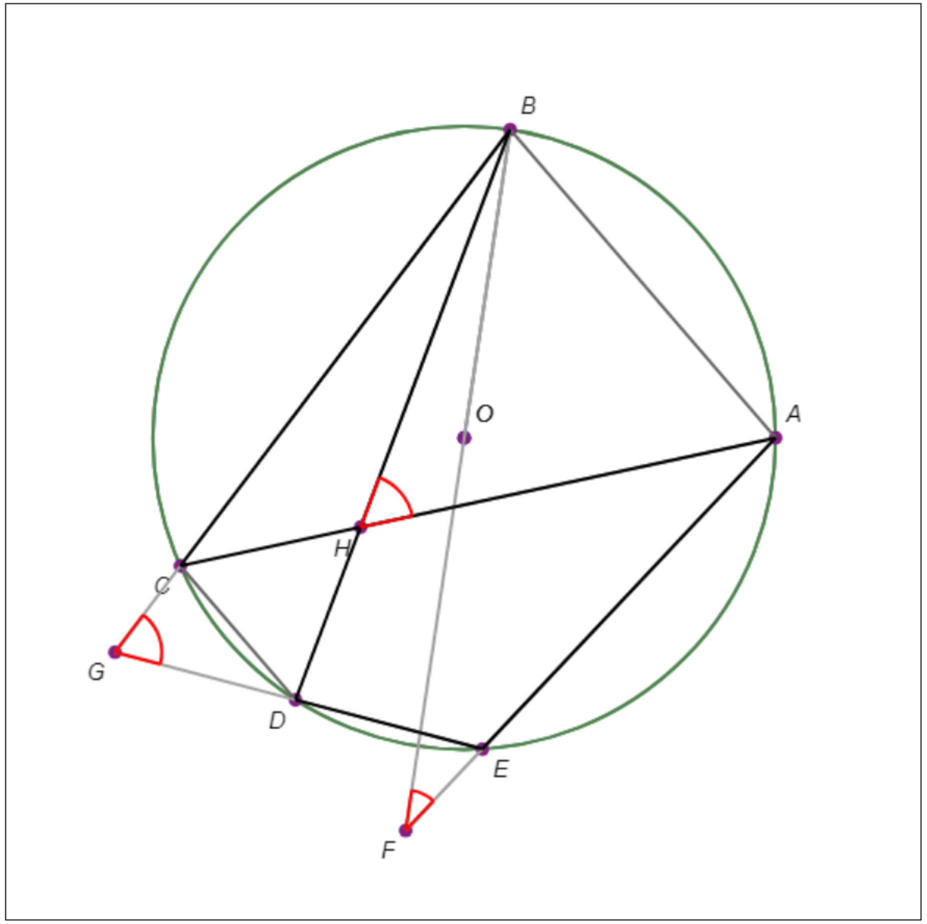
Problem Generator



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(difficulty 27)

Answer | New Problem | Create Collection | Add to Custom Collection

Step by step solution

1. Breadth first find all angles determined by known angles

AHB = x-y+90

Let DBG=w.
As BGD=y, BDG=180-y-w.
As BDG=180-y-w, BDE=y+w.
As BDEA is a cyclic quadrilateral, BAE=180-BDE, so BAE=180-y-w.
As BAF=180-y-w, ABF=y+w-x.
As triangle ABO is isosceles, AOB=2x-2y-2w+180.
As AOB is at the center of a circle on the same chord as ACB, AOB=2ACB, so ACB=x-y-w+90.
As BCH=x-y-w+90, BHC=y-x+90.
As BHC=y-x+90, BHA=x-y+90.

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Step by step solution

AHB = x-y+90

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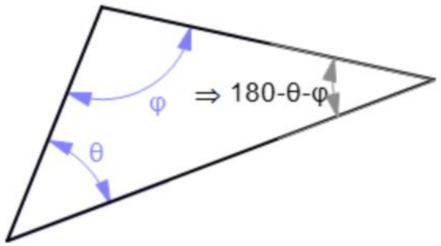
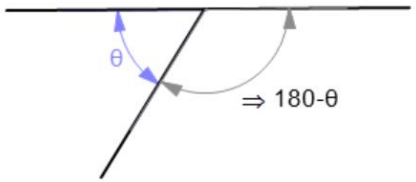
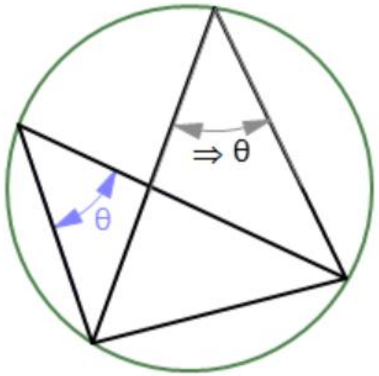
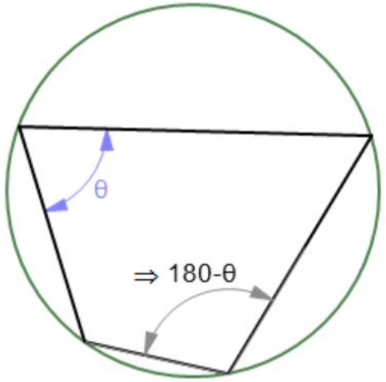
As BCH=x-y-w+90, BHC=y-x+90.
As BHC=y-x+90, BHA=x-y+90.

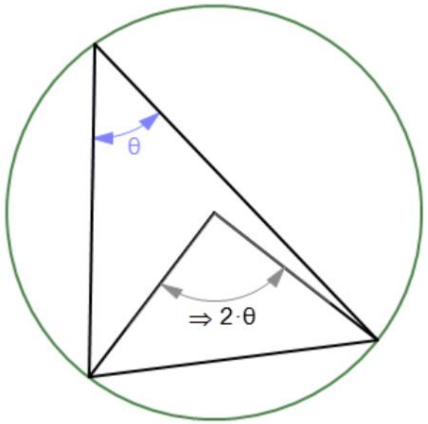
1. Breadth first find all angles determined by known angles using these rules

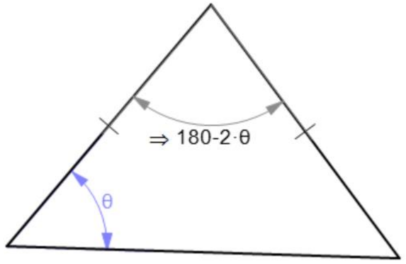












Step by step solution

1. Breadth first find all angles determined by known angles
2. If one of the new angles is known, we can set the known value equal to the new value to obtain a linear equation

AHB = x-y+90

Let DBG=w.
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As BCH=x-y-w+90, BHC=y-x+90.
As BHC=y-x+90, BHA=x-y+90.

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Step by step solution

1. Breadth first find all angles determined by known angles
2. If one of the new angles is known, we can set the known value equal to the new value to obtain a linear equation
3. If not, go back to 1

AHB = x-y+90

Let DBG=w.
As BGD=y, BDG=180-y-w.
As BDG=180-y-w, BDE=y+w.
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As BHC=y-x+90, BHA=x-y+90.

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Estimate difficulty

1. Assign a difficulty value to each step of a solution
2. Sum up the difficulties of the steps

AHB = x-y+90

Let DBG=w.
As BGD=y, BDG=180-y-w.
As BDG=180-y-w, BDE=y+w.
As BDEA is a cyclic quadrilateral, BAE=180-BDE, so BAE=180-y-w.
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As BHC=y-x+90, BHA=x-y+90.

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Problem Set Generator

- Generate problems with step by step solutions and difficulty
- Ensure that no two are isomorphic
- Sort by difficulty

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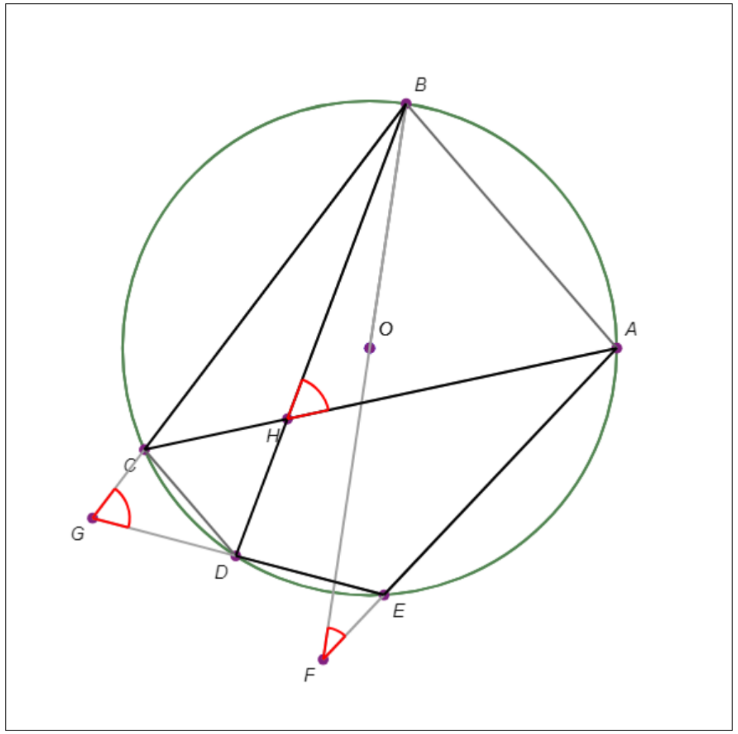
Problem Generator



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(difficulty 27)

Answer | New Problem | Create Collection | Add to Custom Collection

▶ Problem Settings
▼ Collection Settings

Number of Examples: 20

File Name examples.pdf

- [ ] Shuffle
- [ ] Introduction
- [x] Answers
- [ ] Identifiers
- [x] Solutions

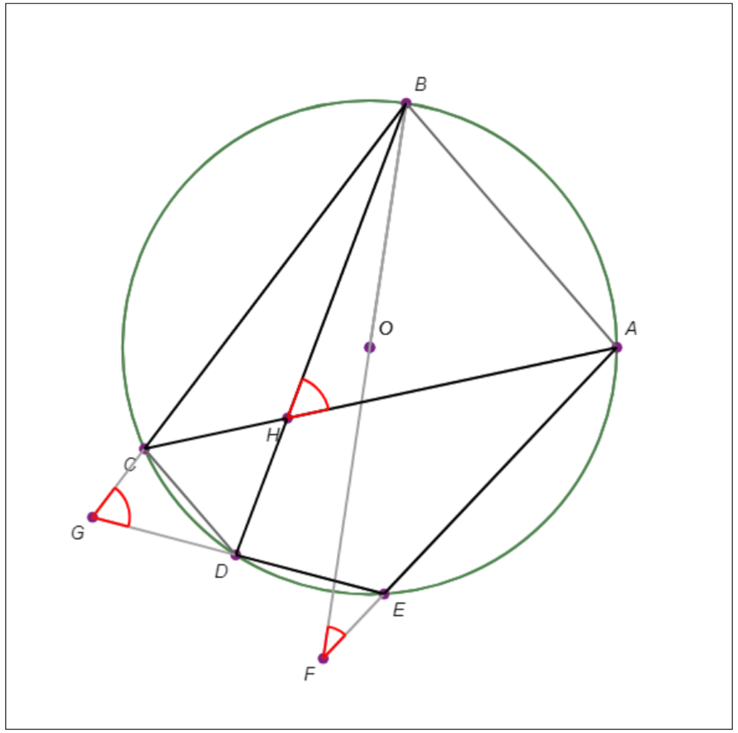
solution for one in 2

Running header my collection

▶ Custom Collection

Application

- Test examples for GXWeb



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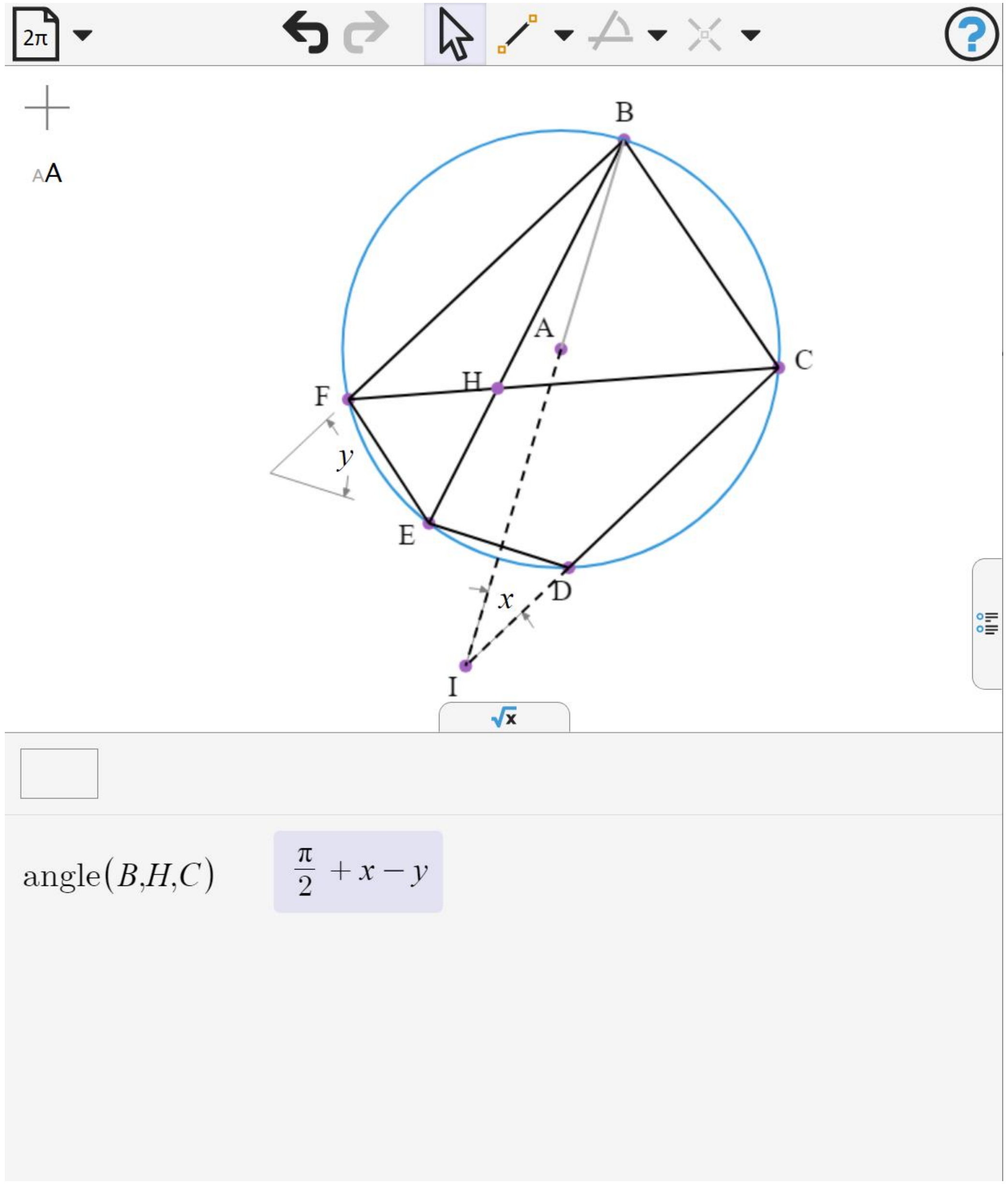
(difficulty 27)

Answer | New Problem | Create Collection | Add to Custom Collection



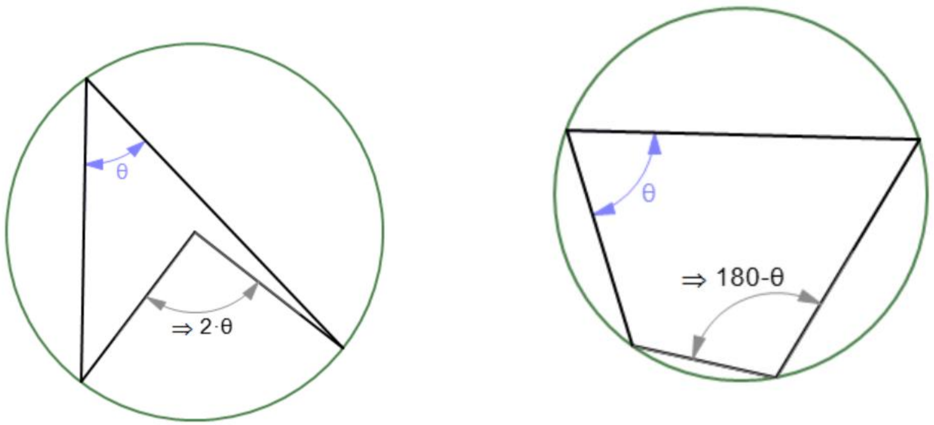
Application

- Problem books

Conclusion

I’d claim these types of naïve angle problems have these benefits for automated problem generation

- Only linear algebra is required in the course of their solution.
- Permuting vertices on a polygon yields problems which are different when solved using the tools of elementary geometry.



- All problems may be solved by the application of a small number of basic theorems.
- The application is not purely rote, and requires some ingenuity.