Coq/GeoCoq cheat sheets

Logic	Coq's syntax
false	`False`
true	`True`
$a = b$	`a = b`
$a \neq b$	`a <> b`
not A	`~ A`
A or B	`A \/ B`
A and B	`A /\ B`
A implies B	`A -> B`
A is equivalent to B	`A <-> B`
$f(x, y, z)$	`(f x y z)`
$\forall xy, P(x, y)$	`forall (x:A) (y:B), P x y`
$\exists xy, P(x, y)$	`exists (x:A) (y:B), P x y`

When the goal is ...	use tactic ...
`p /\ q`	`split`
`p \/ q`	`left` or `right`
`p -> q`	`intro H`
`~p`	`intro H`
`p <-> q`	`split`
`forall x, p`	`intro x`
`exists x, p`	`exists t`
an assumption	`assumption`
a definition	`unfold`

<table>
<tr><th>To use hypothesis H ...</th><th>use tactic ...</th></tr>
<tr><td>p \/ q</td><td>destruct H as [H1|H2]</td></tr>
<tr><td>p /\ q</td><td>destruct H as [H1 H2]</td></tr>
<tr><td>p -> q</td><td>apply H</td></tr>
<tr><td>p <-> q</td><td>apply H</td></tr>
<tr><td>~p</td><td>apply H</td></tr>
<tr><td>False</td><td>contradiction</td></tr>
<tr><td>forall x, p</td><td>apply H or apply H in</td></tr>
<tr><td>exists x, p</td><td>destruct H as [x G]</td></tr>
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To introduce a new hypothesis H ...	use tactic ...
`assert (Hnew: stm).`	
`assert (Hnew:= proof).`	

Adhoc Tactics for Geometry	
Add collinearity	`assert_cols`
Add betweeness	`assert_bets`
Add inequalities	`assert_diffs`
Deduce equalities	`treat_equalities`
Preudo-transitivity of Col	`ColR`
Assumption modulo permutations	`finish`
Assumption modulo permutations and pseudo-transitivity of Col	`sfinish`
apply a lemma modulo permutation of hypotheses	`perm_apply`

Coq	Notation	Explanation	Definition
`Bet A B C`	A—B—C	points A, B and C are collinear and B is between A and C, it can be the case the $A = B$ or $B = C$.	
`Cong A B C D`	$AB \equiv CD$	the segments AB and CD are congruent	
`Col A B C`	$\mathrm{Col}\ A\,B\,C$	points A, B and C are collinear	A—B—$C \vee B$—A—$C \vee A$—C—B
`Out O A B`	$O \not\!\!-\!\!- A \leftrightarrow B$	B belongs to the half line OA	$O \neq A \wedge O \neq B \wedge (O$—$A$—$B \vee O$—$B$—$A)$
`Midpoint M A B`	$A -\!\!+\!\!- M -\!\!+\!\!- B$	M is the midpoint of segment AB	A—M—$B \wedge AM \equiv BM$
`TS A B P Q`	$A \underset{P}{\overset{Q}{-\!\!/\!\!/\!\!-}} B$	P and Q are on different sides of line AB	$\neg\,\mathrm{Col}\ P\,A\,B \wedge \neg\,\mathrm{Col}\ Q\,A\,B \wedge \exists T, \mathrm{Col}\ T\,A\,B \wedge P$—$T$—$Q$
`OS A B X Y`	$A \underset{XY}{-\!\!/\!\!/\!\!-} B$	X and Y are on the same side of line AB	$\exists Z, A \underset{X}{\overset{Z}{-\!\!/\!\!/\!\!-}} B \wedge A \underset{Y}{\overset{Z}{-\!\!/\!\!/\!\!-}} B$
`Coplanar A B C D`	$\mathrm{Cp}\ A\,B\,C\,D$	A, B, C and D belong to the same plane	$\exists X, (\mathrm{Col}\ A\,B\,X \wedge \mathrm{Col}\ C\,D\,X) \vee (\mathrm{Col}\ A\,C\,X \wedge \mathrm{Col}\ B\,D\,X) \vee (\mathrm{Col}\ A\,D\,X \wedge \mathrm{Col}\ B\,C\,X)$
`Concyclic A B C D`		A, B, C and D belong to the same circle	$Coplanar ABCD \wedge \exists O\ OA \equiv OB \wedge OA \equiv OC \wedge OA \equiv OD$
`Per A B C`	$\text{◺}\,A\,B\,C$	the triangle ABC is a right triangle in B	$\exists C', C -\!\!+\!\!- B -\!\!+\!\!- C' \wedge AC \equiv AC'$
`Perp_at P A B C D`	$AB \underset{P}{\perp} CD$	$AB \perp CD$ and P is the intersection of AB and CD	$A \neq B \wedge C \neq D \wedge \mathrm{Col}\ P\,A\,B \wedge \mathrm{Col}\ P\,C\,D \wedge (\forall U\,V, \mathrm{Col}\ U\,A\,B \Rightarrow \mathrm{Col}\ V\,C\,D \Rightarrow \text{◺}\,U\,P\,V)$
`Perp A B C D`	$AB \perp CD$	line AB is perpendicular to line CD	$\exists P, AB \underset{P}{\perp} CD$
`Par_strict A B C D Y`	$AB \parallel_s CD$	line AB is parallel to line CD and $AB \neq CD$	$A \neq B \wedge C \neq D \wedge \mathrm{Cp}\ A\,B\,C\,D \wedge \neg\exists X, \mathrm{Col}\ X\,A\,B \wedge \mathrm{Col}\ X\,C\,D$
`Par A B C D`	$AB \parallel CD$	line AB is parallel to line CD	$AB \parallel_s CD \vee (A \neq B \wedge C \neq D \wedge \mathrm{Col}\ A\,C\,D \wedge \mathrm{Col}\ B\,C\,D)$
`Perp2 A B C D P`	$AB \underset{P}{\perp\!\!\!\perp} CD$	the line AB and CD have a common perpendicular through P	$\exists X, \exists Y, \mathrm{Col}\ P\,X\,Y \wedge XY \perp AB \wedge XY \perp CD$
`Le A B C D`	$AB \leq CD$	the length AB is smaller or equal to length CD	$\exists E, C$—Y—$D \wedge AB \equiv CE$
`Lt A B C D`	$AB < CD$	the length AB is smaller to length CD	$AB \leq CD \wedge \neg AB \equiv CD$
`Ge A B C D`	$AB \geq CD$	the length AB is greater or equal to length CD	$CD \leq AB$
`Gt A B C D`	$AB > CD$	the length AB is greater than length CD	$CD < AB$
`CongA A B C D E F`	$A\,B\,C \mathrel{\widehat{=}} D\,E\,F$	the angles $\angle ABC$ and $\angle DEF$ are congruent	$A \neq B \wedge C \neq B \wedge D \neq E \wedge F \neq E \wedge$ $\exists A', \exists C', \exists D', \exists F', B$—$A$—$A' \wedge AA' \equiv ED \wedge$ B—C—$C' \wedge CC' \equiv EF \wedge E$—$D$—$D' \wedge DD' \equiv BA \wedge$ E—F—$F' \wedge FF' \equiv BC \wedge A'C' \equiv D'F'$
`InAngle P A B C`	$P \mathrel{\widehat{\in}} A\,B\,C$	the point P is inside the angle $\angle ABC$	$A \neq B \wedge C \neq B \wedge P \neq B \wedge \exists X, A$—$X$—$C \wedge$ $(X = B \vee B \not\!\!-\!\!- X \leftrightarrow P)$

Coq	Notation	Explanation	Definition
`LeA A B C D E F`	$A\,B\,C \mathrel{\hat{\leq}} D\,E\,F$	the angle $\angle ABC$ is smaller or equal than angle $\angle DEF$	$\exists P, P \mathrel{\hat{\in}} D\,E\,F \wedge A\,B\,C \mathrel{\hat{=}} D\,E\,P$
`LtA A B C D E F`	$A\,B\,C \mathrel{\hat{<}} D\,E\,F$	the angle $\angle ABC$ is smaller than angle $\angle DEF$	$A\,B\,C \mathrel{\hat{\leq}} D\,E\,F \wedge \neg A\,B\,C \mathrel{\hat{=}} D\,E\,F$
`GtA A B C D E F`	$A\,B\,C \mathrel{\hat{<}} D\,E\,F$	the angle $\angle ABC$ is greater than angle $\angle DEF$	$D\,E\,F \mathrel{\hat{<}} A\,B\,C$
`GeA A B C D E F`	$A\,B\,C \mathrel{\hat{\leq}} D\,E\,F$	the angle $\angle ABC$ is greater than angle $\angle DEF$	$D\,E\,F \mathrel{\hat{\leq}} A\,B\,C$
`Acute A B C`		$\angle ABC$ is an acute angle	$\exists A', \exists B', \exists C', \llcorner A'\,B'\,C' \wedge A\,B\,C \mathrel{\hat{<}} A'\,B'\,C'$
`Obtuse A B C`		$\angle ABC$ is an obtuse angle	$\exists A', \exists B', \exists C', \llcorner A'\,B'\,C' \wedge A'\,B'\,C' \mathrel{\hat{<}} A\,B\,C$
`SuppA A B C`		the angles $\angle ABC$ and $\angle DEF$ are supplementary	$A \neq B \wedge \exists A', A{-}B{-}A' \wedge D\,E\,F \mathrel{\hat{=}} C\,B\,A'$
`SumA A B C D E F G H I`	$A\,B\,C \mathbin{\hat{+}} D\,E\,F \mathrel{\hat{=}} G\,H\,I$	The sum of angles $\angle ABC$ and $\angle DEF$ is congruent to $\angle GHI$	$\exists J \quad C\,B\,J \mathrel{\hat{=}} D\,E\,F \wedge \neg B \underset{AJ}{\not\!-\!-} C \wedge \mathrm{Cp}\ A\,B\,C\,J \wedge A\,B\,J \mathrel{\hat{=}} G\,H\,I$
`SAMS A B C D E F`		The sum of the angles $\angle ABC$ and $\angle DEF$ is smaller than the flat angle.	$A \neq B \wedge (Out E D F \vee \neg A{-}B{-}C) \wedge \exists J\ C\,B\,J \mathrel{\hat{=}} D\,E\,F \wedge \neg B \underset{AJ}{\not\!-\!-} C \wedge \neg A \overset{J}{\underset{C}{\not\!-\!-}} B \wedge \mathrm{Cp}\ A\,B\,C\,J$
`TriSumA A B C D E F`	$\mathcal{S}(\triangle ABC) \mathrel{\hat{=}} D\,E\,F$	The sum of the angles of the triangle ABC is congruent to the angle $\angle DEF$	$\exists G\,H\,I \quad SumA \quad A\,B\,C\,B\,C\,A\,G\,H\,I \wedge SumA\,G\,H\,I\,C\,A\,B\,D\,E\,F$
`isosceles A B C`		ABC is an isosceles triangle in B	$AB \equiv BC$
`equilateral A B C`		ABC is an equilateral triangle	$AB \equiv BC \wedge BC \equiv CA$
`equilateral_strict A B C`		ABC is an equilateral triangle and the points are distinct and hence not collinear	$equilateral\,ABC \wedge A \neq B$
`Cong_3 A B C A' B' C'`		ABC is congruent to $A'B'C'$	$AB \equiv A'B' \wedge AC \equiv A'C' \wedge BC \equiv B'C'$
`CongA_3 A B C A' B' C'`		ABC is similar to $A'B'C'$	$A\,B\,C \mathrel{\hat{=}} A'\,B'\,C' \wedge B\,C\,A \mathrel{\hat{=}} B'\,C'\,A' \wedge C\,A\,B \mathrel{\hat{=}} C'\,A'\,B'$
`is_orthocenter H A B C`		H is the ortho-center of triangle ABC.	
`is_circumcenter G A B C`		G is the circum-center of triangle ABC.	
`is_gravity_center H A B C`		H is the gravity center of triangle ABC.	
`ReflectL P' P A B`		P' is the image of P by reflection on line AB	$(\exists X\ X{-}P{-}P' \wedge \mathrm{Col}\ A\,B\,X) \wedge (AB \perp PP' \vee P = P')$
`Reflect P' P A B`		P' is the image of P by reflection on line AB if $A \neq B$ and P' is the image of P by the reflection on point A if $A = B$	$(A \neq B \wedge ReflectL\,P'\,P\,A\,B) \vee (A = B \wedge A{-}P{-}P')$
`Perp_bisect P Q A B`		PQ is the perpendicular bisector of segment AB	$ReflectL\,A\,B\,P\,Q \wedge A \neq B$
`Orth_at X A B C U V`		$ABC \perp UV$ and X is the intersection of ABC and UV	$\neg\,\mathrm{Col}\ A\,B\,C \wedge U \neq V \wedge \wedge \mathrm{Cp}\ A\,B\,C\,X \wedge \mathrm{Col}\ U\,V\,X \wedge (\forall P\,Q, \mathrm{Cp}\ A\,B\,C\,P \Rightarrow \mathrm{Col}\ U\,V\,Q \Rightarrow \llcorner P\,X\,Q)$
`Orth A B C U V`	$ABC \perp UV$	plane ABC is perpendicular to line UV	$\exists X, Orth\_at\,X\,A\,B\,C\,U\,V$

Coq	Notation	Explanation	Definition
`Parallelogram A B C D`		$ABCD$ is a parallelogram, this includes a flat case defined as diagonals intersect in their midpoint	$Parallelogram\_strict\,A\,B\,A'\,B' \vee Parallelogram\_flat\,A\,B\,A'\,B'$
`Parallelogram_strict A B C D`		$ABCD$ is a parallelogram. The points are not collinear	$A \underset{B B'}{-\!/\!/\!-} A' \;\wedge\; AB \parallel A'B' \;\wedge\; AB \equiv A'B'$
`Parallelogram_flat A B C D`		$ABCD$ is a flat parallelogram	$\text{Col}\ A\,B\,A' \wedge \text{Col}\ A\,B\,B' \wedge AB \equiv A'B' \wedge AB' \equiv A'B \wedge (A \neq A' \vee B \neq B')$
`Saccheri A B C D`		$ABCD$ is a quadrilateral with two equal sides perpendicular to the base. In Euclidean geometry it is a rectangle.	$\text{◺}\,B\,A\,D \wedge \text{◺}\,A\,D\,C \wedge AB \equiv CD \wedge A \underset{B C}{-\!/\!/\!-} D$
`Lambert A B C D`		$ABCD$ is a quadrilateral with three right angles. In hyperbolic geometry the fourth angle is acute, in Euclidean geometry it is a right angle.	$A \neq B \wedge B \neq C \wedge C \neq D \wedge A \neq D \wedge \text{◺}\,B\,A\,D \wedge \text{◺}\,A\,D\,C \wedge \text{◺}\,A\,B\,C$
`Rectangle A B C D`		$ABCD$ is a rectangle	
`Square A B C D`		$ABCD$ is a square	
`Rhombus A B C D`		$ABCD$ is a rhombus	
`Kite A B C D`		$ABCD$ is a kite	

Construction	Coq
three non collinear points	`lower_dim_ex`
two distinct points	`two_distinct_points`
a point X on line AB such that B is on segment AX	`point_construction_different A B`
a point different from A	`another_point A`
a point on the half-line AB at a given distance CD from B	`segment_construction A B C D`
a point on the half-line AB at a given distance CD from A	`segment_construction_2 A B C D`
a point not on the line AB	`not_col_exists A B`
a point on the line AB different from A and B	`diff_coll_ex A B`
another point on the line formed by three collinear points ABC	`diff_col_ex3 A B C`
a point on the opposite side of A wrt. line PQ	`l9_10 P Q A`
a point at the intersection of two perpendicular lines	Definition of Perp
the foot of perpendicular to AB through P	`l8_18_existence A B P`
a point on the perpendicular to AB through A on the opposite side of C	`l8_21 A B C`
the midpoint of segment AB	`midpoint_existence A B`
the symmetric of A wrt. I	`symmetric_point_construction A I`
the symmetric of X wrt. line AB	`ex_sym A B X`
two points on the parallel to line AB through P	`parallel_existence A B P`
a point on the parallel to line AB through P	`parallel_existence_spec A B P`
the circumcenter of triangle ABC	`exists_circumcenter A B C`
the in-center of triangle ABC	`incenter_exists A B C`
the center of gravity of triangle ABC	`is_gravity_center_exist_unique A B C`
the projection of P on line AB in the direction XY	`project_existence P A B X Y`